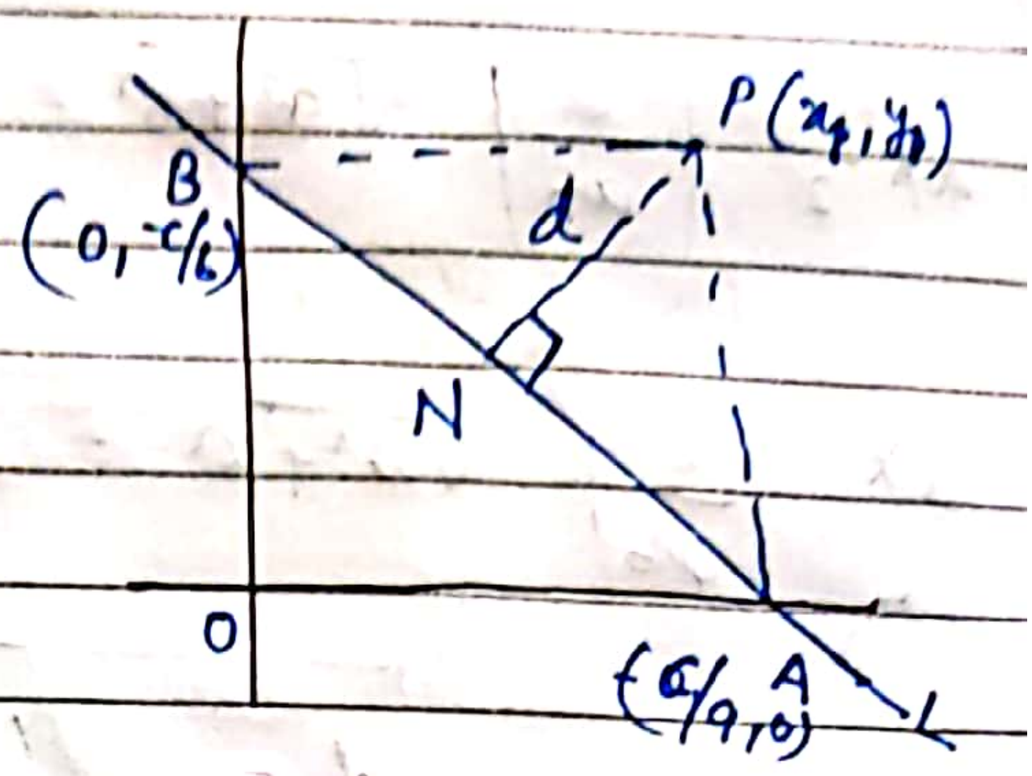
Distance of a point from a line.

Le $ax + by + c = 0$

$\Rightarrow \dfrac{x}{(-c/a)} + \dfrac{y}{(-c/b)} = 1$



$\therefore \quad OA = -\dfrac{c}{a}$

$OB = -\dfrac{c}{b}$

$AB = \sqrt{\left(-\dfrac{c}{a} - 0\right)^2 + \left(-\dfrac{c}{b} - 0\right)^2}$

$= \sqrt{\dfrac{c^2}{a^2} + \dfrac{c^2}{b^2}} = \left|\dfrac{c}{ab}\right| \sqrt{a^2 + b^2} \quad - (i)$

Let $PN = d$

$\therefore$ Area of $\Delta PAB = \dfrac{1}{2} \times AB \times PN$

$= \dfrac{1}{2} \left|\dfrac{c}{ab}\right| \sqrt{a^2 + b^2} \times d$

Area of $\Delta PAB = \dfrac{1}{2} \left| x_1\left(0 + \dfrac{c}{b}\right) + \left(\dfrac{-c}{a}\right)\left(\dfrac{-c}{b} - y_1\right) + 0(y_1 - 0) \right|$

$= \dfrac{1}{2} \left| \dfrac{cx_1}{b} + \dfrac{c^2}{ab} + \dfrac{cy_1}{a} \right| \quad - (ii)$

From (1) & (II)

$$d \times \frac{1}{2} \left| \frac{c}{ab} \right| \sqrt{a^2+b^2} = \frac{1}{2} \left| \frac{cx_1}{a} + \frac{c^2}{ab} + \frac{cy_1}{a} \right|$$

$$d \times \left| \frac{c}{ab} \right| \sqrt{a^2+b^2} = \left| \frac{c}{ab} \right| \sqrt{ax_1 + by_1 + c} \times \text{eq}$$

$$\boxed{d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}}$$

Q) Find the distance b/w the origin & line $3x + 4y - 10 = 0$

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$$

$$= \frac{|3(0) + 4(0) - 10|}{\sqrt{3^2+4^2}}$$

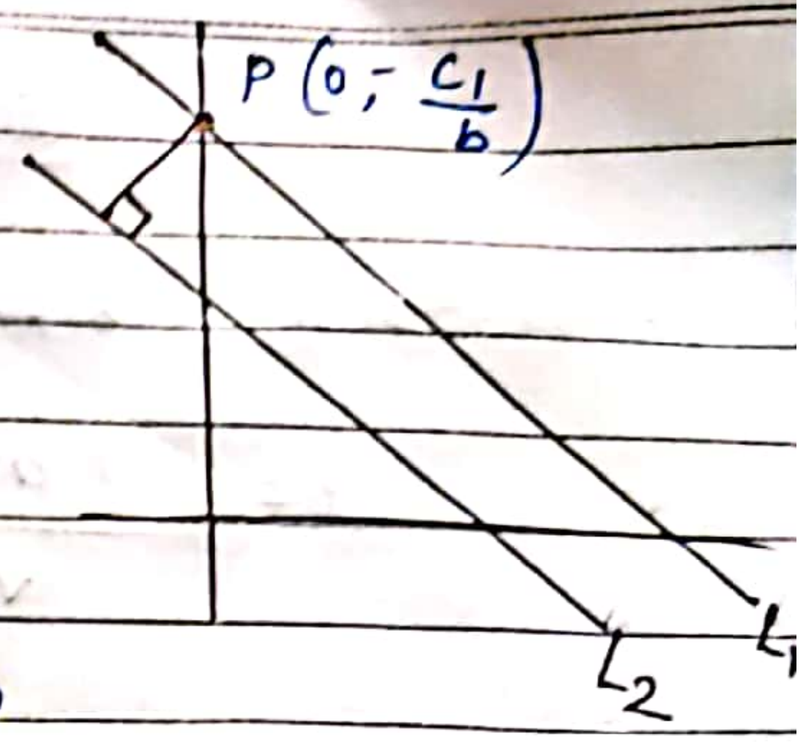
$$= \frac{10}{5} = 2 \text{ unit.}$$

Distance between 2 parallel lines -

$$L_1 \equiv ax_1 + by + c_1 = 0 \quad \text{— (i)}$$
$$L_2 \equiv ax + by + c_2 = 0 \quad \text{— ii)}$$

Put $x = 0$ in eq (1)

$by + c_1 = 0 \Rightarrow y = -\frac{c_1}{b}$



distance of point P from line $L_2 \equiv ax + by + c_2 = 0$

$$d = \left| \frac{(a \times 0) + b \times (-c_1/b) + c_2}{\sqrt{a^2 + b^2}} \right|$$

$$d = \left| \frac{-c_1 + c_2}{\sqrt{a^2 + b^2}} \right|$$

$$\boxed{d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}}$$

Q) Find distance b/w lines $3x + 5y - 15 = 0$ & $6x + 10y - 11 = 0$

$\downarrow$

$6x + 10y - 30 = 0$

$$d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|-30 - (-11)|}{\sqrt{6^2 + 10^2}} = \frac{19}{\sqrt{136}} = \frac{19}{2\sqrt{34}}$$