The sides of a rhombus $ABCD$ are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1, 2)$ and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is

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(a) 2

(b) $\frac{7}{4}$

(c) $\frac{7}{2}$

(d) $\frac{5}{2}$

(d) Let the coordinate A be $(0, c)$

Equations of the given lines are

$x - y + 2 = 0$ and

$7x - y + 3 = 0$

We know that the diagonals of the rhombus will be parallel to the angle bisectors of the two given lines; $y = x + 2$ and $y = 7x + 3$

$\therefore$ equation of angle bisectors is given as:

$$\frac{x - y + 2}{\sqrt{2}} = \pm \frac{7x - y + 3}{5\sqrt{2}}$$

$5x - 5y + 10 = \pm (7x - y + 3)$

$\therefore$ Parallel equations of the diagonals are $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$\therefore$ slopes of diagonals are $\frac{-1}{2}$ and 2.

Now, slope of the diagonal from $A(0, c)$ and passing through $P(1, 2)$ is $(2 - c)$

$\therefore 2 - c = 2 \Rightarrow c = 0$ (not possible)

$\therefore 2 - c = \frac{-1}{2} \Rightarrow c = \frac{5}{2}$

$\therefore$ ordinate of A is $\frac{5}{2}$.