Problem 3:

A series *RLC* circuit with $R = 10.0\ \Omega$, $L = 400$ mH and $C = 2.0\ \mu\text{F}$ is connected to an AC voltage source $V(t) = V_0 \sin \omega t$ which has a maximum amplitude $V_0 = 100$ V.

(a) What is the resonant frequency ω_0?

(b) Find the rms current at resonance.

(c) Let the driving frequency be $\omega = 4000$ rad/s. Assume the current response is given by $I(t) = I_0 \sin(\omega t - \phi)$. Calculate the amplitude of the current and the phase shift between the current and the driving voltage.

Problem 3 Solutions:

(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(400\,\text{mH})(2.0\,\mu\text{F})}} = \frac{1}{\sqrt{8.0\times10^{-7}\text{s}}} = 1.1\times10^{3}\ \text{rad/s}$$

(b)

At resonance, $Z = R$. Therefore,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{(V_0/\sqrt{2})}{R} = \frac{(100V/\sqrt{2})}{10.0\Omega} = 7.07\text{A}$$

(c)

$$X_C = \frac{1}{\omega C} = \frac{1}{(4000\,\text{rad/s})(2.0\mu\text{F})} = 125\,\Omega$$

$$X_L = \omega L = (4000\,\text{rad/s})(400\,\text{mH}) = 1600\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(10.0\Omega)^2 + (1600\,\Omega - 125\,\Omega)^2} = 1475\,\Omega$$

So

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{100\ \text{V}}{\sqrt{(10.0\Omega)^2 + (1600\,\Omega - 125\,\Omega)^2}} = \frac{100\ \text{V}}{1475\,\Omega} = 6.8\times10^{-2}\ \text{A}$$

$$\phi = \tan\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{1600 - 125}{10.0}\right) = \tan^{-1}(147.5) = 89.6°$$