If a function $f(x)$ defined by

$$f(x)=\begin{cases} ae^x+be^{-x}, & -1\le x<1 \\ cx^2, & 1\le x\le 3 \\ ax^2+2cx, & 3<x\le 4 \end{cases}$$ be continuous for some $a, b, c \in \mathbf{R}$

and $f'(0)+f'(2)=e$, then the value of a is :

[Main Sep. 02, 2020 (I)]

(a) $\dfrac{1}{e^2-3e+13}$

(b) $\dfrac{e}{e^2-3e-13}$

(c) $\dfrac{e}{e^2+3e+13}$

(d) $\dfrac{e}{e^2-3e+13}$

(d) Since, function $f(x)$ is continuous at $x = 1, 3$

$\therefore f(1)=f(1^+) \Rightarrow ae+be^{-1}=c$ ...(i)

$f(3)=f(3^+)$

$\Rightarrow 9c=9a+6c \Rightarrow c=3a$ ...(ii)

From (i) and (ii),

$b=ae(3-e)$ ...(iii)

$$f'(x)=\begin{cases} ae^x-be^{-x} & -1<x<1 \\ 2cx & 1<x<3 \\ 2ax+2c & 3<x<4 \end{cases}$$

$f'(0)=a-b,\ f'(2)=4c$

Given, $f'(0)+f'(2)=e$

$a-b+4c=e$ ...(iv)

From eqs. (i), (ii), (iii) and (iv),

$a-3ae+ae^2+12a=e$

$\Rightarrow 13a-3ae+ae^2=e \Rightarrow a=\dfrac{e}{e^2-3e+13}$