7.30 For an LCR circuit driven at frequency ω, the equation reads $L \frac{di}{dt} + Ri + \frac{q}{C} = v_i = v_m \sin \omega t$

(i) Multiply the equation by i and simplify where possible.

(ii) Interpret each term physically.

(iii) Cast the equation in the form of a conservation of energy statement.

(iv) Integrate the equation over one cycle to find that the phase difference between v and i must be acute.

Answer:

$L\ di/dt + Ri + q/C = vi = vm \sin \omega t$

i) Multiplying the above equation with I, we get

$d(1/2\ Li^2)/dt + 1/2C\ dq^2/dt + i^2R/2 = ½\ Vm\ i \sin \omega t$

ii) $d(1/2\ Li^2)/dt$ represents the rate of change of potential energy in inductance L

$d/dt\ q^2/2C$ represents the energy stored in dt time in the capacitor

i^2R represents the joules heating loss

$½\ Vm\ i \sin \omega t$ is the rate of driving force

iii) The first equation is in the form of conservation of energy

iv) Integrating the equation from o to T we get dt as positive which is possible when the phase difference is constant and the angle made is acute.