One kmol of any isotope contains Avogadro's number of atoms, and so 1g of ${}^{238}_{92}\text{U}$ contains

$$\frac{10^{-3}}{238}\text{kmol} \times 6.025 \times 10^{26} \text{ atoms/kmol}$$

= 25.3×10^{20} atoms.

The decay rate R is

$R = \lambda N$

$$= \frac{0.693}{T_{1/2}} N = \frac{0.693 \times 25.3 \times 10^{20}}{1.42 \times 10^{17}} s^{-1}$$

$= 1.23 \times 10^{4}\ \text{s}^{-1}$

$= 1.23 \times 10^{4}$ Bq

EXAMPLE 13.4

Example 13.5 Tritium has a half-life of 12.5 y undergoing beta decay. What fraction of a sample of pure tritium will remain undecayed after 25 y.

Solution

By definition of half-life, half of the initial sample will remain undecayed after 12.5 y. In the next 12.5 y, one-half of these nuclei would have decayed. Hence, one fourth of the sample of the initial pure tritium will remain undecayed.

EXAMPLE 13.5

13.6.2 Alpha decay

A well-known example of alpha decay is the decay of uranium ${}^{238}_{92}\text{U}$ to thorium ${}^{234}_{90}\text{Th}$ with the emission of a helium nucleus ${}^{4}_{2}\text{He}$

$$ {}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} \qquad (\alpha\text{-decay}) \qquad (13.19)$$

In α-decay, the mass number of the product nucleus (daughter nucleus) is four less than that of the decaying nucleus (parent nucleus), while the atomic number decreases by two. In general, α-decay of a parent nucleus ${}^{A}_{Z}\text{X}$ results in a daughter nucleus ${}^{A-4}_{Z-2}\text{Y}$

$$ {}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He} \qquad (13.20)$$

From Einstein's mass-energy equivalance relation [Eq. (13.6)] and energy conservation, it is clear that this spontaneous decay is possible only when the total mass of the decay products is less than the mass of the initial nucleus. This difference in mass appears as kinetic energy of the products. By referring to a table of nuclear masses, one can check that the total mass of ${}^{234}_{90}\text{Th}$ and ${}^{4}_{2}\text{He}$ is indeed less than that of ${}^{238}_{92}\text{U}$.

The disintegration energy or the Q-value of a nuclear reaction is the difference between the initial mass energy and the total mass energy of the decay products. For α-decay

$$Q = (m_{X} - m_{Y} - m_{He})\, c^{2} \qquad (13.21)$$

Q is also the net kinetic energy gained in the process or, if the initial nucleus X is at rest, the kinetic energy of the products. Clearly, $Q > 0$ for exothermic processes such as α-decay.

Example 13.6 We are given the following atomic masses:

$^{238}_{92}\text{U}$ = 238.05079 u $^{4}_{2}\text{He}$ = 4.00260 u

$^{234}_{90}\text{Th}$ = 234.04363 u $^{1}_{1}\text{H}$ = 1.00783 u

$^{237}_{91}\text{Pa}$ = 237.05121 u

Here the symbol Pa is for the element protactinium (Z = 91).

(a) Calculate the energy released during the alpha decay of $^{238}_{92}\text{U}$.

(b) Show that $^{238}_{92}\text{U}$ can not spontaneously emit a proton.

Solution

(a) The alpha decay of $^{238}_{92}\text{U}$ is given by Eq. (13.20). The energy released in this process is given by

$Q = (m_U - m_{Th} - m_{He})\, c^2$

Substituting the atomic masses as given in the data, we find

$Q = (238.05079 - 234.04363 - 4.00260)\text{u} \times c^2$

$= (0.00456 \text{ u})\, c^2$

$= (0.00456 \text{ u})\,(931.5 \text{ MeV/u})$

$= 4.25$ MeV.

(b) If $^{238}_{92}\text{U}$ spontaneously emits a proton, the decay process would be

$^{238}_{92}\text{U} \rightarrow {}^{237}_{91}\text{Pa} + {}^{1}_{1}\text{H}$

The Q for this process to happen is

$= (m_U - m_{Pa} - m_H)\, c^2$

$= (238.05079 - 237.05121 - 1.00783)\text{ u} \times c^2$

$= (-\,0.00825 \text{ u})\, c^2$

$= -\,(0.00825 \text{ u})(931.5 \text{ MeV/u})$

$= -\,7.68$ MeV

Thus, the Q of the process is negative and therefore it cannot proceed spontaneously. We will have to supply an energy of 7.68 MeV to a $^{238}_{92}\text{U}$ nucleus to make it emit a proton.

EXAMPLE 13.6

13.6.3 Beta decay

In beta decay, a nucleus spontaneously emits an electron (β^- decay) or a positron (β^+ decay). A common example of β^- decay is

$$^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + e^- + \bar{\nu} \qquad (13.22)$$

and that of β^+ decay is

$$^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + e^+ + \nu \qquad (13.23)$$

The decays are governed by the Eqs. (13.14) and (13.15), so that one can never predict *which* nucleus will undergo decay, but one can characterize the decay by a half-life $T_{1/2}$. For example, $T_{1/2}$ for the decays above is respectively 14.3 d and 2.6y. The emission of electron in β^- decay is accompanied by the emission of an antineutrino ($\bar{\nu}$); in β^+ decay, instead, a neutrino (ν) is generated. Neutrinos are neutral particles with very small (possiblly, even zero) mass compared to electrons. They have only weak interaction with other particles. They are, therefore, very difficult to detect, since they can penetrate large quantity of matter (even earth) without any interaction.

In both β^- and β^+ decay, the mass number A remains unchanged. In β^- decay, the atomic number Z of the nucleus goes up by 1, while in β^+ decay Z goes down by 1. The basic nuclear process underlying β^- decay is the conversion of neutron to proton

$$n \to p + e^- + \bar{\nu} \qquad (13.24)$$

while for β^+ decay, it is the conversion of proton into neutron

$$p \to n + e^+ + \nu \qquad (13.25)$$

Note that while a free neutron decays to proton, the decay of proton to neutron [Eq. (13.25)] is possible only inside the nucleus, since proton has smaller mass than neutron.

13.6.4 Gamma decay

Like an atom, a nucleus also has discrete energy levels - the ground state and excited states. The scale of energy is, however, very different. Atomic energy level spacings are of the order of eV, while the difference in nuclear energy levels is of the order of MeV. When a nucleus in an excited state spontaneously decays to its ground state (or to a lower energy state), a photon is emitted with energy equal to the difference in the two energy levels of the nucleus. This is the so-called *gamma decay*. The energy (MeV) corresponds to radiation of extremely short wavelength, shorter than the hard X-ray region.

Typically, a gamma ray is emitted when a α or β decay results in a daughter nucleus in an excited state. This then returns to the ground state by a single photon transition or successive transitions involving more than one photon. A familiar example is the successive emmission of gamma rays of energies 1.17 MeV and 1.33 MeV from the deexcitation of $^{60}_{28}Ni$ nuclei formed from β^- decay of $^{60}_{27}Co$.

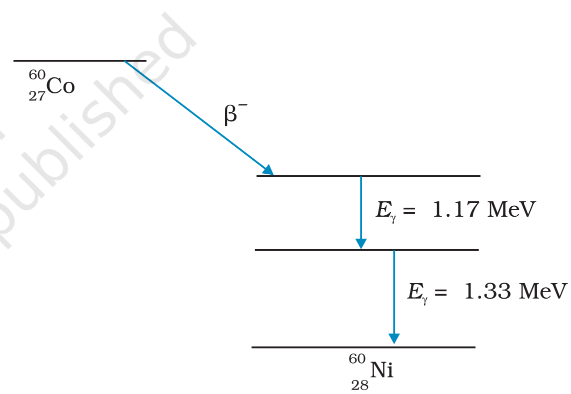

FIGURE 13.4 β-decay of $^{60}_{28}Ni$ nucleus followed by emission of two γ rays from deexcitation of the daughter nucleus $^{60}_{28}Ni$.

13.7 Nuclear Energy

The curve of binding energy per nucleon E_{bn}, given in Fig. 13.1, has a long flat middle region between A = 30 and A = 170. In this region the binding energy per nucleon is nearly constant (8.0 MeV). For the lighter nuclei region, A < 30, and for the heavier nuclei region, A > 170, the binding energy per nucleon is less than 8.0 MeV, as we have noted earlier. Now, the greater the binding energy, the less is the total mass of a bound system, such as a nucleus. Consequently, if nuclei with less total binding energy transform to nuclei with greater binding energy, there will be a net energy release. This is what happens when a heavy nucleus decays into two or more intermediate mass fragments (*fission*) or when light nuclei fuse into a havier nucleus (*fusion*.)

Exothermic chemical reactions underlie conventional energy sources such as coal or petroleum. Here the energies involved are in the range of