M_{11} is the *self-inductance* and is written as L_1. Therefore,

$$\varepsilon_1 = -L_1 \frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$$

Example 6.10 (a) Obtain the expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid. (b) How does this magnetic energy compare with the electrostatic energy stored in a capacitor?

Solution

(a) From Eq. (6.19), the magnetic energy is

$$U_B = \frac{1}{2}LI^2$$

$$= \frac{1}{2}L\left(\frac{B}{\mu_0 n}\right)^2 \quad \text{(since } B = \mu_0 nI, \text{ for a solenoid)}$$

$$= \frac{1}{2}(\mu_0 n^2 Al)\left(\frac{B}{\mu_0 n}\right)^2 \quad \text{[from Eq. (6.17)]}$$

$$= \frac{1}{2\mu_0}B^2 Al$$

(b) The magnetic energy per unit volume is,

$$u_B = \frac{U_B}{V} \quad \text{(where } V \text{ is volume that contains flux)}$$

$$= \frac{U_B}{Al}$$

$$= \frac{B^2}{2\mu_0} \tag{6.20}$$

We have already obtained the relation for the electrostatic energy stored per unit volume in a parallel plate capacitor (refer to Chapter 2, Eq. 2.77),

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \tag{2.77}$$

In both the cases energy is proportional to the square of the field strength. Equations (6.20) and (2.77) have been derived for special cases: a solenoid and a parallel plate capacitor, respectively. But they are general and valid for any region of space in which a magnetic field or/and an electric field exist.

EXAMPLE 6.10

PHYSICS

Interactive animation on ac generator:
http://micro.magnet.fsu.edu/electromag/java/generator/ac.html

6.10 AC Generator

The phenomenon of electromagnetic induction has been technologically exploited in many ways. An exceptionally important application is the generation of alternating currents (ac). The modern ac generator with a typical output capacity of 100 MW is a highly evolved machine. In this section, we shall describe the basic principles behind this machine. The Yugoslav inventor Nicola Tesla is credited with the development of the machine. As was pointed out in Section 6.3, one method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area. As the coil rotates in a magnetic field **B**, the effective area of the loop (the face perpendicular to the field) is *A* cos *θ*, where *θ* is the angle between **A** and **B**. This method of producing a flux change is the principle of operation of a simple ac generator. An ac generator converts mechanical energy into electrical energy.

The basic elements of an ac generator are shown in Fig. 6.16. It consists of a coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil (called armature) is mechanically rotated in the uniform magnetic field by some external means. The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings and brushes.

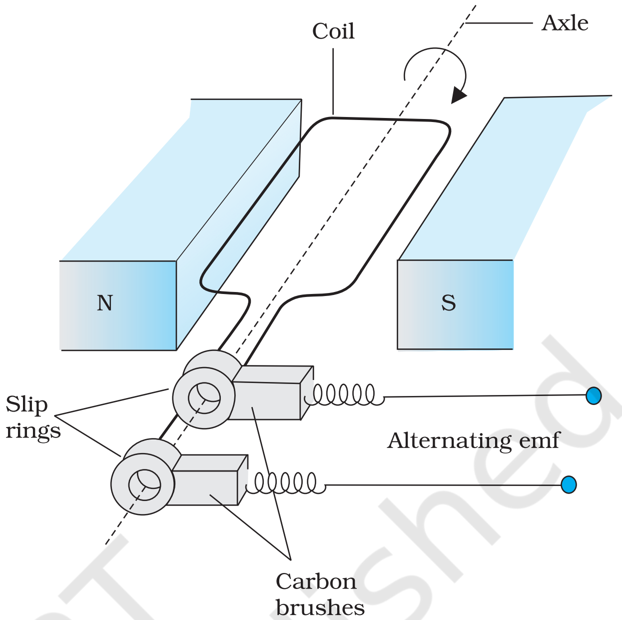

FIGURE 6.16 AC Generator

When the coil is rotated with a constant angular speed *ω*, the angle *θ* between the magnetic field vector **B** and the area vector **A** of the coil at any instant *t* is $\theta = \omega t$ (assuming $\theta = 0°$ at $t = 0$). As a result, the effective area of the coil exposed to the magnetic field lines changes with time, and from Eq. (6.1), the flux at any time *t* is

$$\Phi_B = BA \cos \theta = BA \cos \omega t$$

From Faraday's law, the induced emf for the rotating coil of *N* turns is then,

$$\varepsilon = -N \frac{d\Phi_B}{dt} = -NBA \frac{d}{dt}(\cos \omega t)$$

Thus, the instantaneous value of the emf is

$$\varepsilon = NBA\,\omega \sin \omega t \tag{6.21}$$

where *NBAω* is the maximum value of the emf, which occurs when $\sin \omega t = \pm 1$. If we denote *NBAω* as ε_0, then

$$\varepsilon = \varepsilon_0 \sin \omega t \tag{6.22}$$

Since the value of the sine fuction varies between +1 and –1, the sign, or polarity of the emf changes with time. Note from Fig. 6.17 that the emf has its extremum value when $\theta = 90°$ or $\theta = 270°$, as the change of flux is greatest at these points.

The direction of the current changes periodically and therefore the current is called *alternating current* (ac). Since $\omega = 2\pi\nu$, Eq (6.22) can be written as

$$\varepsilon = \varepsilon_0 \sin 2\pi \nu t \tag{6.23}$$

where *ν* is the frequency of revolution of the generator's coil.

Note that Eq. (6.22) and (6.23) give the instantaneous value of the emf and *ε* varies between $+\varepsilon_0$ and $-\varepsilon_0$ periodically. We shall learn how to determine the time-averaged value for the alternating voltage and current in the next chapter.

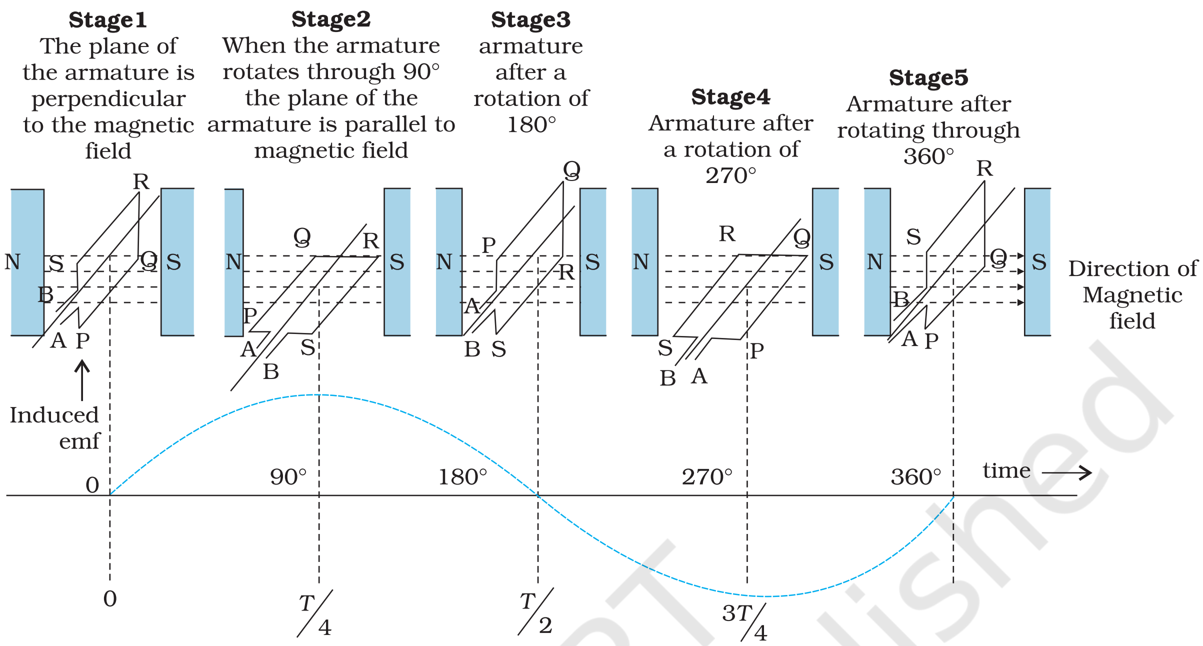

FIGURE 6.17 An alternating emf is generated by a loop of wire rotating in a magnetic field.

In commercial generators, the mechanical energy required for rotation of the armature is provided by water falling from a height, for example, from dams. These are called *hydro-electric generators*. Alternatively, water is heated to produce steam using coal or other sources. The steam at high pressure produces the rotation of the armature. These are called *thermal generators*. Instead of coal, if a nuclear fuel is used, we get *nuclear power generators*. Modern day generators produce electric power as high as 500 MW, i.e., one can light up 5 million 100 W bulbs! In most generators, the coils are held stationary and it is the electromagnets which are rotated. The frequency of rotation is 50 Hz in India. In certain countries such as USA, it is 60 Hz.

Example 6.11 Kamla peddles a stationary bicycle. The pedals of the bicycle are attached to a 100 turn coil of area 0.10 m^2. The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil?

Solution Here $\nu = 0.5$ Hz; $N = 100$, $A = 0.1$ m^2 and $B = 0.01$ T. Employing Eq. (6.21)

$$\varepsilon_0 = NBA\,(2\,\pi\,\nu)$$

$$= 100 \times 0.01 \times 0.1 \times 2 \times 3.14 \times 0.5$$

$$= 0.314 \text{ V}$$

The maximum voltage is 0.314 V.

We urge you to explore such alternative possibilities for power generation.

EXAMPLE 6.11