

Exemplar Problem with Solution :

For a loaded die, the probabilities of outcomes are given as under:

$$P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1 \text{ and } P(4) = 0.3.$$

The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more', respectively. Determine whether or not A and B are independent.

Soln :

For a loaded die, it is given that

$$P(1) = P(2) = 0.2, \\ P(3) = P(5) = P(6) = 0.1 \text{ and } P(4) = 0.3$$

Also, die is thrown two times.

Here, A = Same number each time and B = Total score is 10 or more

$$\therefore A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$\begin{aligned} \text{So, } P(A) &= [P(1, 1) + P(2, 2) + P(3, 3) + P(4, 4) + P(5, 5) + P(6, 6)] \\ &= [P(1) \cdot P(1) + P(2) \cdot P(2) + P(3) \cdot P(3) + P(4) \cdot P(4) + P(5) \cdot P(5) + P(6) \cdot P(6)] \\ &= [0.2 \times 0.2 + 0.2 \times 0.2 + 0.1 \times 0.1 + 0.3 \times 0.3 + 0.1 \times 0.1 + 0.1 \times 0.1] \\ &= 0.04 + 0.04 + 0.01 + 0.09 + 0.01 + 0.01 = 0.20 \end{aligned}$$

$$\text{and } B = \{(4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$$

$$\begin{aligned} \therefore P(B) &= P(4, 6) + P(6, 4) + P(5, 5) + P(5, 6) + P(6, 5) + P(6, 6) \\ &= P(4) \cdot P(6) + P(6) \cdot P(4) + P(5) \cdot P(5) + P(5) \cdot P(6) + P(6) \cdot P(5) + P(6) \cdot P(6) \\ &= 0.3 \times 0.1 + 0.1 \times 0.3 + 0.1 \times 0.1 + 0.1 \times 0.1 + 0.1 \times 0.1 + 0.1 \times 0.1 \\ &= 0.03 + 0.03 + 0.01 + 0.01 + 0.01 + 0.01 = 0.10 \end{aligned}$$

$$\text{Also, } A \cap B = \{(5, 5), (6, 6)\}$$

$$\begin{aligned} \therefore P(A \cap B) &= P(5, 5) + P(6, 6) = P(5) \cdot P(5) + P(6) \cdot P(6) \\ &= 0.1 \times 0.1 + 0.1 \times 0.1 = 0.01 + 0.01 = 0.02 \end{aligned}$$

We know that, for two events A and B, if $P(A \cap B) = P(A) \cdot P(B)$, then both are independent events.

$$\text{Here, } P(A \cap B) = 0.02 \text{ and } P(A) \cdot P(B) = 0.20 \times 0.10 = 0.02$$

$$\text{Thus, } P(A \cap B) = P(A) \cdot P(B) = 0.02$$

Hence, A and B are independent events.