Exemplar Problem with Solution :

For a loaded die, the probabilities of outcomes are given as under:

P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1 and P(4) = 0.3.

Let A and B be the events, 'same number each

time', and 'a total score is 10 or more', respectively. Determine whether or not

A and B are independent.

If the die were fair, determine whether or not the

events A and B are independent.

Soln :

Referring to the above solution, we have

$$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$\Rightarrow$ $n(A) = 6$ and $n(S) = 6^2 = 36$ [where, S is sample space]

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

and $B = \{(4, 6), (6, 4), (5, 5), (6, 5), (5, 6), (6, 6)\}$

$\Rightarrow$ $n(B) = 6$ and $n(S) = 6^2 = 36$

$\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Also, $A \cap B = \{(5, 5), (6, 6)\}$

$\Rightarrow$ $n(A \cap B) = 2$ and $n(S) = 36$

$\therefore$ $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Also, $P(A) \cdot P(B) = \frac{1}{36}$

Thus, $P(A \cap B) \neq P(A) \cdot P(B)$ $\left[\because \frac{1}{18} \neq \frac{1}{36}\right]$

So, we can say that both A and B are not independent events.