

Exemplar Problem with Solution :

A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$.

Find :

- (i) $P(A|B)$ (ii) $P(B|A)$ (iii) $P(A'|B)$ (iv) $P(A'|B')$

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3}, \quad P(A \cap B) = \frac{1}{4}$$

$$(i) \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

$$(ii) \quad P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2}$$

$$(iii) \quad P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}} = \frac{\frac{3}{12} - \frac{1}{4}}{\frac{1}{3}} = \frac{\frac{3}{12} - \frac{3}{12}}{\frac{1}{3}} = \frac{0}{\frac{1}{3}} = 0$$

$$(iv) \quad P(\bar{A}|\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(B)} = \frac{1 - [\frac{1}{2} + \frac{1}{3} - \frac{1}{4}]}{1 - \frac{1}{3}} = \frac{10/12}{2/3} = \frac{5}{8}$$