

Q6. Let  $0 < \alpha < \frac{\pi}{2}$  be fixed angle. If

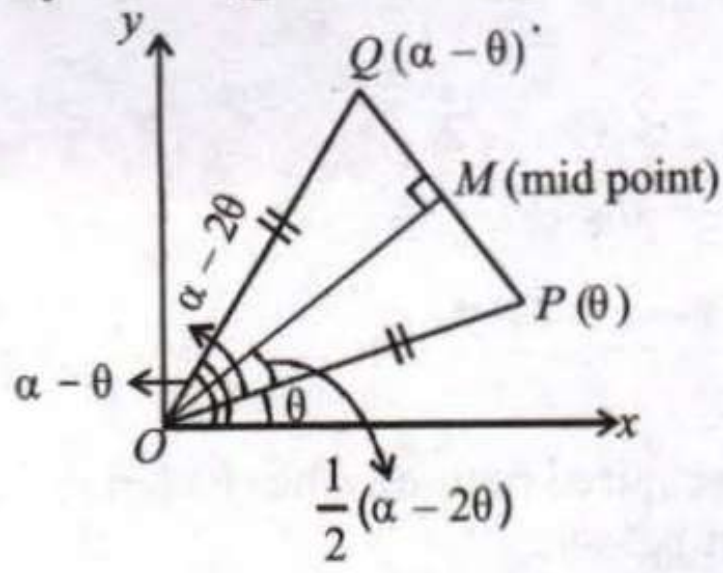
$P = (\cos \theta, \sin \theta)$  and  $Q = (\cos(\alpha - \theta), \sin(\alpha - \theta))$ ,

then  $Q$  is obtained from  $P$  by

**[2002S]**

- (a) clockwise rotation around origin through an angle  $\alpha$
- (b) anticlockwise rotation around origin through an angle  $\alpha$
- (c) reflection in the line through origin with slope  $\tan \alpha$
- (d) reflection in the line through origin with slope  $\tan (\alpha / 2)$

Sol 6. (d) Clearly  $OP = OQ = 1$  and  $\angle QOP = \alpha - \theta - \theta = \alpha - 2\theta$ .



The bisector of  $\angle QOP$  will be a perpendicular bisector of  $PQ$  also. Hence  $Q$  is reflection of  $P$  in the line  $OM$  which makes an angle  $\angle MOP + \angle POX$  with  $x$ -axis, i.e.,

$$\frac{1}{2}(\alpha - 2\theta) + \theta = \alpha/2.$$

So that slope of  $OM$  is  $\tan \alpha/2$ .