

Q5. Two equal sides of an isosceles triangle are given by the equations  $7x - y + 3 = 0$  and  $x + y - 3 = 0$  and its third side passes through the point  $(1, -10)$ . Determine the equation of the third side. **[1984 - 4 Marks]**

Sol 5.

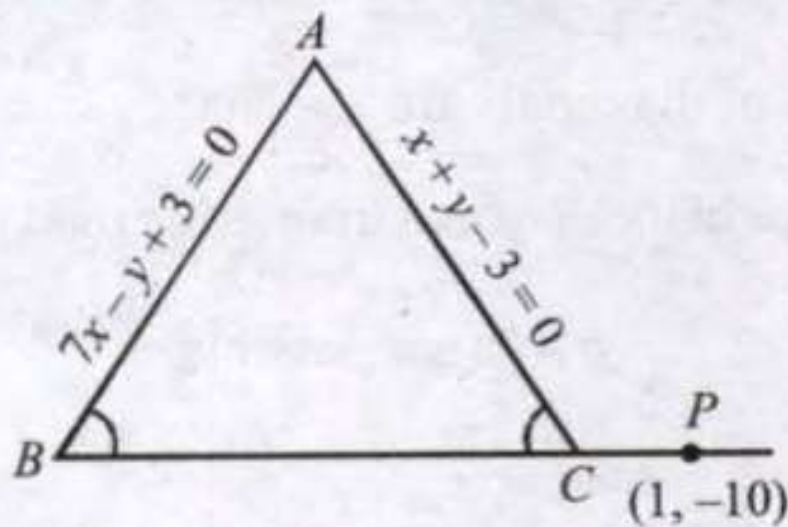
Let equations of equal sides  $AB$  and  $AC$  of isosceles  $\triangle ABC$  are

$$7x - y + 3 = 0$$

and  $x + y - 3 = 0$

Now slope of  $AB = 7$  and slope of  $AC = -1$

The third side  $BC$  of the triangle passes through the point  $(1, -10)$ . Let its slope be  $m$ .



As  $AB = AC$

$\therefore \angle B = \angle C$

$\rightarrow \tan B = \tan C$

$$\Rightarrow \frac{7-m}{1+7m} = \pm \left( \frac{-1-m}{1-m} \right)$$

On taking '+' sign, we get

$$(7-m)(1-m) = -(1+m)(1+7m)$$

$$\Rightarrow 7 - 8m + m^2 + 7m^2 + 8m + 1 = 0$$

$$\Rightarrow 8m^2 + 8 = 0 \Rightarrow m^2 + 1 = 0$$

It has no real solution.

On taking '-' sign, we get

$$(7-m)(1-m) = (1+m)(1+7m)$$

$$\Rightarrow 7 - 8m + m^2 - 7m^2 - 8m - 1 = 0$$

$$\Rightarrow -6m^2 - 16m + 6 = 0 \Rightarrow 3m^2 + 8m - 3 = 0$$

$$\Rightarrow (3m-1)(m+3) = 0 \Rightarrow m = 1/3, -3$$

$\therefore$  The required line is

$$y + 10 = \frac{1}{3}(x - 1) \text{ or } y + 10 = -3(x - 1)$$

$$\text{i.e. } x - 3y - 31 = 0 \text{ or } 3x + y + 7 = 0.$$