Q4. The base of an equilateral triangle is along the line given by $3x+4y=9$. If a vertex of the triangle is $(1, 2)$, then the length of a side of the triangle is:

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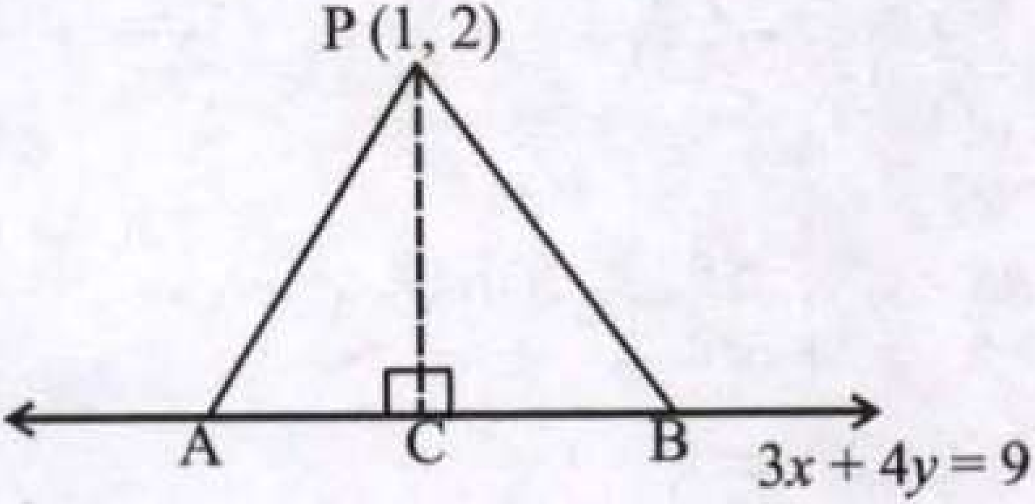
(a) $\frac{2\sqrt{3}}{15}$ (b) $\frac{4\sqrt{3}}{15}$ (c) $\frac{4\sqrt{3}}{5}$ (d) $\frac{2\sqrt{3}}{5}$

Sol 4. **(b)**



Shortest distance of a point (x_1, y_1) from line

$ax + by = c$ is $d = \left|\frac{ax_1 + by_1 - c}{\sqrt{a^2 + b^2}}\right|$

Now shortest distance of P (1, 2) from $3x + 4y = 9$ is

$$PC = d = \left|\frac{3(1) + 4(2) - 9}{\sqrt{3^2 + 4^2}}\right| = \frac{2}{5}$$

Given that ΔAPB is an equilateral triangle

Let 'a' be its side

Now, In ΔPCB, $(PB)^2 = (PC)^2 + (CB)^2$

(By Pythagoras theoresm)

$$a^2 = \left(\frac{2}{5}\right)^2 + \frac{a^2}{4}$$

$$a^2 - \frac{a^4}{4} = \frac{4}{25} \Rightarrow \frac{3a^2}{4} = \frac{4}{25}$$

$$a^2 = \frac{16}{75} \Rightarrow a = \sqrt{\frac{16}{75}} = \frac{4}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{15}$$

$\therefore$ Length of Equilateral triangle $(a) = \dfrac{4\sqrt{3}}{15}$