Q3. Consider a triangle Δ whose two sides lie on the x-axis and the line $x+y+1=0$. If the orthocenter of Δ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle Δ is [Adv. 2021]

(a) $x^2+y^2-3x+y=0$

(b) $x^2+y^2-x+3y=0$

(c) $x^2+y^2+2y-1=0$

(d) $x^2+y^2+x+y=0$

Sol 3. $x+y+1=0 \Rightarrow A(-1,0)$

Let vertex B be $(\alpha, -\alpha-1)$

Line AC $\perp$ BH so, $m_{AC} \cdot m_{BH} = -1$

$$\Rightarrow \quad 0 = -\frac{(1-\alpha)}{\alpha+2} \Rightarrow \alpha = 1 \Rightarrow B(1,-2)$$

Let vertex C be $(\beta, 0)$

Line AH $\perp$ BC

$$\therefore \quad m_{AH} \cdot m_{BC} = -1$$

$$\Rightarrow \quad \frac{1}{2} \cdot \frac{2}{\beta-1} = -1 \Rightarrow \quad \beta = 0$$

Centroid of ΔABC is $\left(0, -\frac{2}{3}\right)$

We know that G (centroid) divides line joining circumcentre (O) and orthocentre (H) in the ratio 1 : 2.

$$\Rightarrow \quad \underset{O}{(h,k)} \quad \underset{G}{\left(0,-\frac{2}{3}\right)} \quad \underset{H}{(1,1)}$$

O — 1 — G — 2 — H

$$2h+1=0 \Rightarrow \frac{2k+1}{3} = -\frac{2}{3}$$

$$\Rightarrow \quad h = -\frac{1}{2} \Rightarrow k = -\frac{3}{2}$$

$$\Rightarrow \quad \text{Circumcentre is } \left(-\frac{1}{2}, -\frac{3}{2}\right).$$

Equation of circum circle is (passing through C (0, 0)) is $x^2+y^2+x+3y=0$