

Q3.

Consider a triangle Δ whose two sides lie on the x -axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle Δ is [Adv. 2021]

(a) $x^2 + y^2 - 3x + y = 0$

(b) $x^2 + y^2 - x + 3y = 0$

(c) $x^2 + y^2 + 2y - 1 = 0$

(d) $x^2 + y^2 + x + y = 0$

Sol 3.

$$x + y + 1 = 0 \Rightarrow A(-1, 0)$$

Let vertex B be $(\alpha, -\alpha - 1)$

Line AC \perp BH so, $m_{AC} \cdot m_{BH} = -1$

$$\Rightarrow 0 = -\frac{(1-\alpha)}{\alpha+2} \Rightarrow \alpha = 1 \Rightarrow B(1, -2)$$

Let vertex C be $(\beta, 0)$

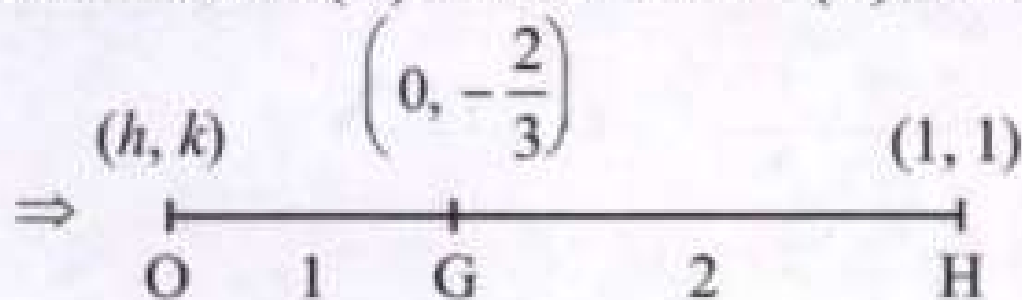
Line AH \perp BC

$$\therefore m_{AH} \cdot m_{BC} = -1$$

$$\Rightarrow \frac{1}{2} \cdot \frac{2}{\beta-1} = -1 \Rightarrow \beta = 0$$

Centroid of ΔABC is $\left(0, -\frac{2}{3}\right)$

We know that G (centroid) divides line joining circumcentre (O) and orthocentre (H) in the ratio 1 : 2.



$$2h + 1 = 0 \Rightarrow \frac{2k + 1}{3} = -\frac{2}{3}$$

$$\Rightarrow h = -\frac{1}{2} \Rightarrow k = -\frac{3}{2}$$

\Rightarrow Circumcentre is $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.

Equation of circum circle is (passing through C (0, 0))
is $x^2 + y^2 + x + 3y = 0$