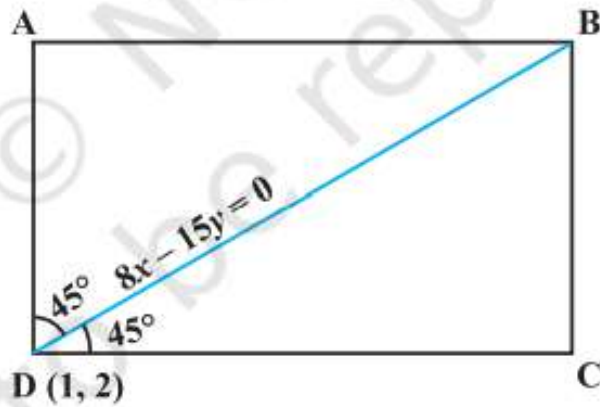**Example** If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at $(1, 2)$, then find the equation of sides of the square passing through this vertex.

Solution Let ABCD be the given square and the coordinates of the vertex D be $(1, 2)$. We are required to find the equations of its sides DC and AD.

Fig. 10.3

Given that BD is along the line $8x - 15y = 0$, so its slope is $\dfrac{8}{15}$ (Why?). The angles made by BD with sides AD and DC is $45°$ (Why?). Let the slope of DC be m. Then

$$\tan 45° = \frac{m - \dfrac{8}{15}}{1 + \dfrac{8m}{15}} \qquad \text{(Why?)}$$

or $$15 + 8m = 15m - 8$$

or $$7m = 23, \text{ which gives } m = \frac{23}{7}$$

Therefore, the equation of the side DC is given by

$$y - 2 = \frac{23}{7} (x - 1) \text{ or } 23x - 7y - 9 = 0.$$

Similarly, the equation of another side AD is given by

$$y - 2 = \frac{-7}{23} (x - 1) \text{ or } 7x + 23y - 53 = 0.$$