Example The reflection of the point (4, -13) about the line 5x + y + 6 = 0 is

$$(A) (-1, -14)$$

$$(C)$$
 $(0,0)$

(D) (1, 2)

Solution The correct choice is (A). Let (h, k) be the point of reflection of the given point (4, -13) about the line 5x+y+6=0. The mid-point of the line segment joining points (h, k) and (4, -13) is given by

$$\frac{h+4}{2}, \frac{k-13}{2} \tag{Why?}$$

This point lies on the given line, so we have

$$5 \frac{h+4}{2} + \frac{k-13}{2} + 6 = 0$$

or

$$5 h + k + 19 = 0 \tag{1}$$

Again the slope of the line joining points (h, k) and (4, -13) is given by $\frac{k+13}{h-4}$. This line

is perpendicular to the given line and hence (-5) $\frac{k+3}{h-4} = -1$ (Why?)

This gives

$$5k + 65 = h - 4$$

or

$$h - 5k - 69 = 0 (2)$$

On solving (1) and (2), we get h = -1 and k = -14. Thus the point (-1, -14) is the reflection of the given point.