

Q3. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is **[Adv. 2014]**

Sol 3.

Let the point P be (x, y) .

$$\text{Then } d_1(P) = \left| \frac{x-y}{\sqrt{2}} \right| \text{ and } d_2(P) = \left| \frac{x+y}{\sqrt{2}} \right|$$

For P lying in first quadrant $x > 0, y > 0$.

$$\text{Now } 2 \leq d_1(P) + d_2(P) \leq 4$$

$$\Rightarrow 2 \leq \left| \frac{x-y}{\sqrt{2}} \right| + \left| \frac{x+y}{\sqrt{2}} \right| \leq 4$$

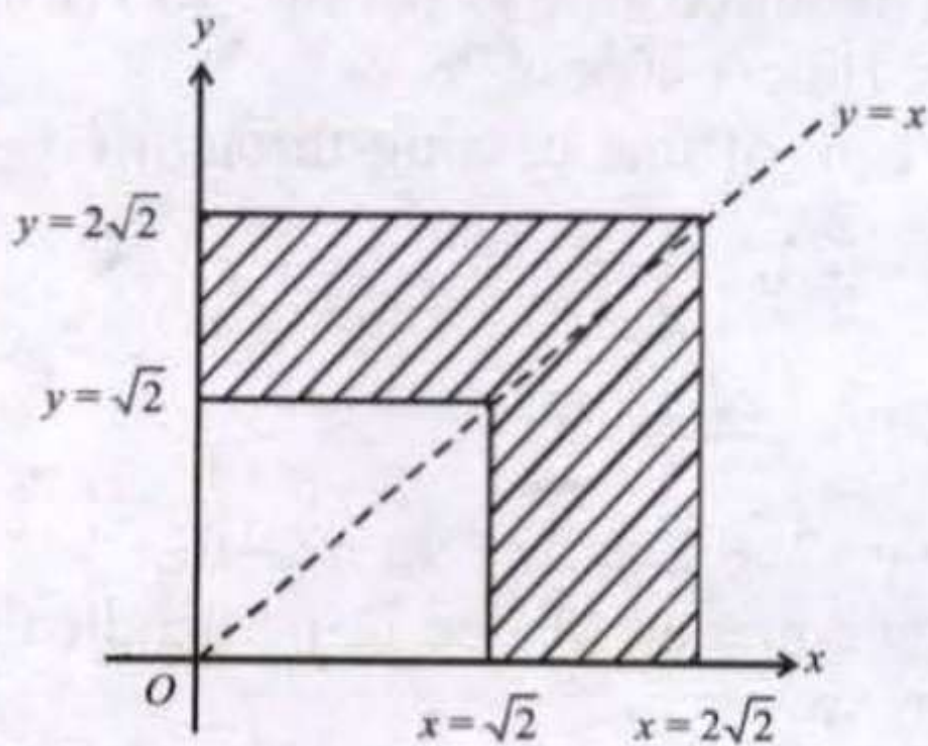
$$\text{If } x > y, \text{ then } 2 \leq \frac{x-y+x+y}{\sqrt{2}} \leq 4$$

$$\Rightarrow \sqrt{2} \leq x \leq 2\sqrt{2}$$

If $x < y$, then

$$2 \leq \frac{y-x+x+y}{\sqrt{2}} \leq 4 \text{ or } \sqrt{2} \leq y \leq 2\sqrt{2}$$

The required region is the shaded region in the figure given below.



$$\therefore \text{Required area} = (2\sqrt{2})^2 - (\sqrt{2})^2 = 8 - 2 = 6 \text{ sq units.}$$