

Example If the slope of a line passing through the point A(3, 2) is $\frac{3}{4}$, then find points on the line which are 5 units away from the point A.

Solution Equation of the line passing through (3, 2) having slope $\frac{3}{4}$ is given by

$$y - 2 = \frac{3}{4} (x - 3)$$

or $4y - 3x + 1 = 0$ (1)

Let (h, k) be the points on the line such that

$$(h - 3)^2 + (k - 2)^2 = 25 \quad (2) \quad (\text{Why?})$$

Also, we have

$$4k - 3h + 1 = 0 \quad (3) \quad (\text{Why?})$$

or

$$k = \frac{3h - 1}{4} \quad (4)$$

Putting the value of k in (2) and on simplifying, we get

$$25h^2 - 150h - 175 = 0 \quad (\text{How?})$$

or

$$h^2 - 6h - 7 = 0$$

or

$$(h + 1)(h - 7) = 0 \Rightarrow h = -1, h = 7$$

Putting these values of k in (4), we get $k = -1$ and $k = 5$. Therefore, the coordinates of the required points are either $(-1, -1)$ or $(7, 5)$.