5) If the system of linear equations

$x+y+3z=0$

$x+3y+k^2z=0$

$3x+y+3z=0$

has a non-zero solution $(x,y,z)$ for some $k \in \mathbb{R}$ then, $x+\left(\frac{y}{z}\right)$ is equal to:

(a) $-3$ (b) 9 (c) 3 (d) $-9$

Solution: (a) Since, system of linear equations has non-zero

$\therefore \Delta = 0$

$$\rightarrow \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 1(9-k^2) - 1(3-3k^2) + 3(1-9) = 0$$

$$\Rightarrow 9-k^2-3+3k^2-24=0$$

$$\Rightarrow 2k^2 = 18 \Rightarrow k^2 = 9, \quad k = \pm 3.$$

So, equations are

$x+y+3z=0$ — (1)

$x+3y+9z=0$ — (2)

$3x+y+3z=0$ — (3)

Now, from equation (1) − (2)

$$-2y-6z=0 \Rightarrow y=-3z \Rightarrow \frac{y}{z}=-3 \quad \text{— (4)}$$

Now, from equation (1) − (3),

$-2x=0 \Rightarrow x=0$

So, $x+\frac{y}{z} = 0-3 = -3$