

4) Let $\lambda \in \mathbb{R}$. The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

[Main Sep. 05, 2020 (I)]

- (a) exactly one negative value of λ
- (b) exactly one positive value of λ
- (c) every value of λ
- (d) exactly two value of λ

Solution:

$$(a) \quad \therefore \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0$$

$$\Rightarrow 3\lambda^2 - 7\lambda - 12 = 0$$

$$\Rightarrow \lambda = 3 \text{ or } -\frac{2}{3}$$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3-\lambda)$$

$$\therefore \text{When } \lambda = -\frac{2}{3}, D_1 \neq 0$$

Hence, equations will be inconsistent when $\lambda = -\frac{2}{3}$.