Q3. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts intercepts of length 8 on these lines.

[1986 - 5 Marks]

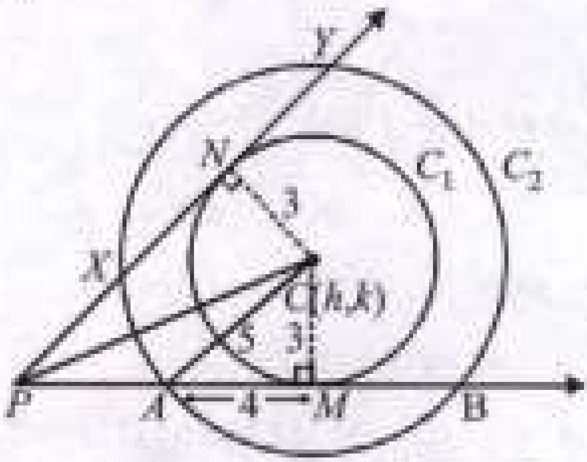
Sol 3. Let equation of tangent PAB be $5x+12y-10=0$ and that of PXY be $5x-12y-40=0$

Now let centre of circles C_1 and C_2 be $C(h, k)$.

Let $CM \perp PAB$, then CM = radius of $C_1 = 3$

Also C_2 makes an intercept of length 8 units on PAB

$\Rightarrow \quad AM = 4$



Now in ΔAMC, we get $AC = \sqrt{4^2+3^2} = 5$

$\therefore \quad$ Radius of C_2 is $= 5$ units

Since $\quad 5x+12y-10=0 \quad$ ...(i)

and $\quad 5x-12y-40=0 \quad$ ...(ii)

are tangents to C_1, therefore length of perpendicular from C to $AB = 3$ units

$$\therefore \quad \frac{5h+12k-10}{13} = \pm 3$$

$\Rightarrow \quad 5h+12k-49=0 \quad$ ...(i)

or $\quad 5h+12k+29=0 \quad$ ...(ii)

$$\text{Similarly} \quad \frac{5h-12k-40}{13} = \pm 3$$

$\Rightarrow \quad 5h-12k-79=0 \quad$ ...(iii)

or $\quad 5h-12k-1=0 \quad$ ...(iv)

Since C lies in first quadrant,

$\therefore \quad h, k$ are $+ve$

$\therefore \quad$ Equation (ii) is not possible.

On solving (i) and (iii), we get

$h = 64/5, k = -5/4$

This is also not possible.

Now solving (i) and (iv), we get $h = 5, k = 2$.

Thus centre of C_2 is $(5, 2)$ and radius 5.

$\therefore \quad$ Equation of C_2 is $(x-5)^2+(y-2)^2 = 5^2$

$\Rightarrow \quad x^2+y^2-10x-4y+4=0$