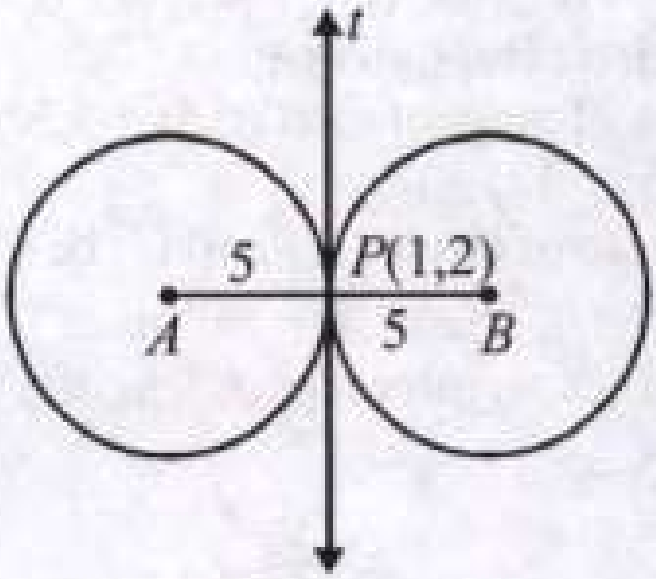
Q.1 Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$, find the equation of the circles. **[1991 - 4 Marks]**

Sol 1. Let t be the common tangent given by

$$4x+3y=10 \qquad \ldots(i)$$

Common point of contact being $P(1,2)$

Let A and B be the centres of the required circles. Clearly, AB is the line perpendicular to t and passing through $P(1,2)$.



$\therefore$ Equation of line AB is

$$\frac{x-1}{4/5}=\frac{x-2}{3/5}=r \left[\begin{array}{l}\text{As slope of } t \text{ is} = -4/3 \\ \therefore \text{slope of } AB \text{ is} = 3/4 = \tan\theta \\ \therefore \cos\theta = 4/5; \sin\theta = 3/5\end{array}\right]$$

For point A, $r=-5$ and for point B, $r=5$, we get

$$\frac{x-1}{4/5}=\frac{y-2}{3/5}=-5,5 \left[\begin{array}{l}\text{Radius of each circle} \\ \text{being } 5, AP=PB=5\end{array}\right]$$

$\Rightarrow$ For point A, $x=-4+1, y=-3+2$

and For point B, $x=4+1, y=3+2$

$\therefore$ $A(-3,-1)$, $B(5,5)$.

$\therefore$ Equation of required circles are

$$(x+3)^2+(y+1)^2=5^2$$

and $(x-5)^2+(y-5)^2=5^2$

$$\Rightarrow \qquad x^2+y^2+6x+2y-15=0$$

$$\text{and } x^2+y^2-10x-10y+25=0$$