JEE Advanced 2018 Paper 1 Offline

MCQ (Single Correct Answer)

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are $\Delta x, \Delta y$ and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$. is $1 \pm (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

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In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first $1.0s$. For $|x| << 1. \ln(1+x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

A 0.04

B 0.03

C 0.02

D 0.01

Explanation

The law of radioactive decay gives number of undecayed nuclei N at time t as $N = N_0 e^{-\lambda t}$, where N_0 is number of nuclei at t = 0 and λ is the decay constant. The number of nuclei decayed till time t is given by $N_d = N_0 - N$. Thus, the decay constant is given by

$\lambda = \frac{1}{t} \ln \frac{N_0}{N} = \frac{1}{t} \ln \frac{N_0}{N_0 - N_d}$.

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Since, N_0 = 3000 and t = 1 s are given without any error, we assume that error in λ is due to error in N_d only.

Hence, we can write above equation as

$$\lambda + \Delta\lambda = \frac{1}{t} \ln \frac{N_0}{N_0 - (N_d + \Delta N_d)}$$

$$= \frac{1}{t} \ln \frac{N_0}{(N_0 - N_d)(1 - \Delta N_d / (N_0 - N_d))}$$

$$= \frac{1}{t} \ln \frac{N_0}{N_0 - N_d} - \frac{1}{t} \ln \left(1 - \frac{\Delta N_d}{N_0 - N_d}\right)$$

$$\approx \lambda + \frac{1}{t} \frac{\Delta N_d}{N_0 - N_d},$$

which gives $\Delta\lambda = \frac{\Delta N_d}{t(N_0 - N_d)}$. Substitute ΔN_d = 40, t = 1 s, and N_d = 1000 to get $\Delta\lambda$ = 0.02.