

(Q. 5) Calculate the row-reduced echelon form of $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix}$

Solution. $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix}$

$$R_2 \rightarrow R_2 + (-1)R_1 \quad \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$R_3 \rightarrow R_3 + (-3)R_1$$

$$R_1 \rightarrow R_1 + (-1)R_2 \quad \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

$$R_3 \rightarrow R_3 + (-1)R_2$$

$$R_3 \rightarrow \left(-\frac{1}{2}\right)R_3 \quad \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_2 \rightarrow R_2 + (-1)R_3 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_1 \rightarrow R_1 + (-1)R_3$$

\therefore The row-reduced echelon form of $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix}$ is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$