(In each operation, take all parameters, other than the one specified, to remain unchanged.)

Solution

(a) Angular separation of the fringes remains constant (= λ/d). The actual separation of the fringes increases in proportion to the distance of the screen from the plane of the two slits.

(b) The separation of the fringes (and also angular separation) decreases. See, however, the condition mentioned in (d) below.

(c) The separation of the fringes (and also angular separation) decreases. See, however, the condition mentioned in (d) below.

(d) Let s be the size of the source and S its distance from the plane of the two slits. For interference fringes to be seen, the condition $s/S < \lambda/d$ should be satisfied; otherwise, interference patterns produced by different parts of the source overlap and no fringes are seen. Thus, as S decreases (i.e., the source slit is brought closer), the interference pattern gets less and less sharp, and when the source is brought too close for this condition to be valid, the fringes disappear. Till this happens, the fringe separation remains fixed.

(e) Same as in (d). As the source slit width increases, fringe pattern gets less and less sharp. When the source slit is so wide that the condition $s/S \le \lambda/d$ is not satisfied, the interference pattern disappears.

(f) The interference patterns due to different component colours of white light overlap (incoherently). The central bright fringes for different colours are at the same position. Therefore, the central fringe is white. For a point P for which $S_2P - S_1P = \lambda_b/2$, where λ_b ($\approx$ 4000 Å) represents the wavelength for the blue colour, the blue component will be absent and the fringe will appear red in colour. Slightly farther away where $S_2Q - S_1Q = \lambda_b = \lambda_r/2$ where λ_r ($\approx$ 8000 Å) is the wavelength for the red colour, the fringe will be predominantly blue.

Thus, the fringe closest on either side of the central white fringe is red and the farthest will appear blue. After a few fringes, no clear fringe pattern is seen.

EXAMPLE 10.4

10.6 Diffraction

If we look clearly at the shadow cast by an opaque object, close to the region of geometrical shadow, there are alternate dark and bright regions just like in interference. This happens due to the phenomenon of diffraction. Diffraction is a general characteristic exhibited by all types of waves, be it sound waves, light waves, water waves or matter waves. Since the wavelength of light is much smaller than the dimensions of most obstacles; we do not encounter diffraction effects of light in everyday observations. However, the finite resolution of our eye or of optical instruments such as telescopes or microscopes is limited due to the phenomenon of diffraction. Indeed the colours that you see when a CD is viewed is due to diffraction effects. We will now discuss the phenomenon of diffraction.

10.6.1 The single slit

In the discussion of Young's experiment, we stated that a single narrow slit acts as a new source from which light spreads out. Even before Young, early experimenters – including Newton – had noticed that light spreads out from narrow holes and slits. It seems to turn around corners and enter regions where we would expect a shadow. These effects, known as *diffraction*, can only be properly understood using wave ideas. After all, you are hardly surprised to hear sound waves from someone talking around a corner!

When the double slit in Young's experiment is replaced by a single narrow slit (illuminated by a monochromatic source), a broad pattern with a central bright region is seen. On both sides, there are alternate dark and bright regions, the intensity becoming weaker away from the centre (Fig. 10.16). To understand this, go to Fig. 10.15, which shows a parallel beam of light falling normally on a single slit LN of width a. The diffracted light goes on to meet a screen. The midpoint of the slit is M.

A straight line through M perpendicular to the slit plane meets the screen at C. We want the intensity at any point P on the screen. As before, straight lines joining P to the different points L,M,N, etc., can be treated as parallel, making an angle θ with the normal MC.

The basic idea is to divide the slit into much smaller parts, and add their contributions at P with the proper phase differences. We are treating different parts of the wavefront at the slit as secondary sources. Because the incoming wavefront is parallel to the plane of the slit, these sources are in phase.

The path difference NP – LP between the two edges of the slit can be calculated exactly as for Young's experiment. From Fig. 10.15,

$$\begin{aligned} NP - LP &= NQ \\ &= a \sin \theta \\ &\approx a\theta \end{aligned} \qquad (10.21)$$

Similarly, if two points M_1 and M_2 in the slit plane are separated by y, the path difference $M_2P - M_1P \approx y\theta$. We now have to sum up equal, coherent contributions from a large number of sources, each with a different phase. This calculation was made by Fresnel using integral calculus, so we omit it here. The main features of the diffraction pattern can be understood by simple arguments.

At the central point C on the screen, the angle θ is zero. All path differences are zero and hence all the parts of the slit contribute in phase. This gives maximum intensity at C. Experimental observation shown in

Fig. 10.15 indicates that the intensity has a central maximum at θ = 0 and other secondary maxima at $\theta \approx (n+1/2)\ \lambda/a$, and has minima (zero intensity) at $\theta \approx n\lambda/a$, $n = \pm1, \pm2, \pm3, ...$ It is easy to see why it has minima at these values of angle. Consider first the angle θ where the path difference $a\theta$ is λ. Then,

$$\theta \approx \lambda / a . \quad (10.22)$$

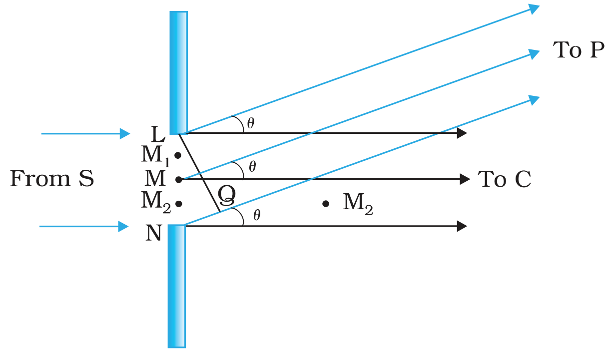


FIGURE 10.15 The geometry of path differences for diffraction by a single slit.

Now, divide the slit into two equal halves LM and MN each of size $a/2$. For every point M_1 in LM, there is a point M_2 in MN such that $M_1M_2 = a/2$. The path difference between M_1 and M_2 at P = $M_2P - M_1P$ = $\theta a/2 = \lambda/2$ for the angle chosen. This means that the contributions from M_1 and M_2 are 180° out of phase and cancel in the direction $\theta = \lambda/a$. Contributions from the two halves of the slit LM and MN, therefore, cancel each other. Equation (10.22) gives the angle at which the intensity falls to zero. One can similarly show that the intensity is zero for $\theta = n\lambda/a$, with n being any integer (except zero!). Notice that the angular size of the central maximum increases when the slit width a decreases.

It is also easy to see why there are maxima at $\theta \simeq (n + 1/2)\ \lambda/a$ and why they go on becoming weaker and weaker with increasing n. Consider an angle $\theta = 3\lambda/2a$ which is midway between two of the dark fringes. Divide the slit into three equal parts. If we take the first two thirds of the slit, the path difference between the two ends would be

$$\frac{2}{3}a \times \theta = \frac{2a}{3} \times \frac{3\lambda}{2a} = \lambda \quad (10.23)$$

The first two-thirds of the slit can therefore be divided into two halves which have a $\lambda/2$ path difference. The contributions of these two halves cancel in the same manner as described earlier. Only the remaining one-third of the slit contributes to the intensity at a point between the two minima. Clearly, this will be much weaker than the central maximum (where the entire slit contributes in phase). One can similarly show that there are maxima at $(n + 1/2)\ \theta/a$ with n = 2, 3, etc. These become weaker with increasing n, since only one-fifth, one-seventh, etc., of the slit contributes in these cases. The photograph and intensity pattern corresponding to it is shown in Fig. 10.16.

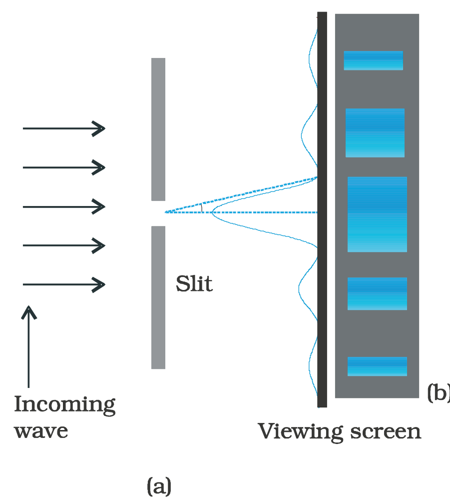


FIGURE 10.16 Intensity distribution and photograph of fringes due to diffraction at single slit.

There has been prolonged discussion about difference between intereference and diffraction among scientists since the discovery of these phenomena. In this context, it is interesting to note what Richard Feynman* has said in his famous Feynman Lectures on Physics:

No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them. The best we can do is, roughly speaking, is to say that when there are only a few sources, say two interfering sources, then the result is usually called interference, but if there is a large number of them, it seems that the word diffraction is more often used.

In the double-slit experiment, we must note that the pattern on the screen is actually a superposition of single-slit diffraction from each slit or hole, and the double-slit interference pattern. This is shown in Fig. 10.17. It shows a broader diffraction peak in which there appear several fringes of smaller width due to double-slit interference. The number of interference fringes occuring in the broad diffraction peak depends on the ratio d/a, that is the ratio of the distance between the two slits to the width of a slit. In the limit of a becoming very small, the diffraction pattern will become very flat and we will obsrve the two-slit interference pattern [see Fig. 10.13(b)].

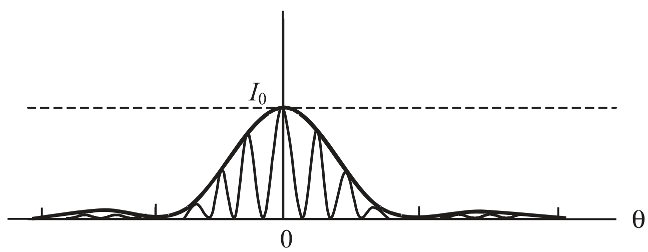


FIGURE 10.17 The actual double-slit interference pattern. The envelope shows the single slit diffraction.

Interactive animation on single slit diffraction pattern
http://www.phys.hawaii.edu/~teb/optics/java/slitdiffr/

PHYSICS

EXAMPLE 10.5

Example 10.5 In Example 10.3, what should the width of each slit be to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern?

Solution We want $a\theta = \lambda, \theta = \dfrac{\lambda}{a}$

$$10\frac{\lambda}{d} = 2\frac{\lambda}{a} \quad a = \frac{d}{5} = 0.2 \text{ mm}$$

Notice that the wavelength of light and distance of the screen do not enter in the calculation of a.

In the double-slit interference experiment of Fig. 10.12, what happens if we close one slit? You will see that it now amounts to a single slit. But you will have to take care of some shift in the pattern. We now have a source at S, and only one hole (or slit) S_1 or S_2. This will produce a single-

* Richard Feynman was one of the recipients of the 1965 Nobel Prize in Physics for his fundamental work in quantum electrodynamics.

slit diffraction pattern on the screen. The centre of the central bright fringe will appear at a point which lies on the straight line SS_1 or SS_2, as the case may be.

We now compare and contrast the interference pattern with that seen for a coherently illuminated single slit (usually called the single slit diffraction pattern).

(i) The interference pattern has a number of equally spaced bright and dark bands. The diffraction pattern has a central bright maximum which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre, on either side.

(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.

(iii) For a single slit of width a, the first null of the interference pattern occurs at an angle of λ/a. At the same angle of λ/a, we get a maximum (not a null) for two narrow slits separated by a distance a.

One must understand that both d and a have to be quite small, to be able to observe good interference and diffraction patterns. For example, the separation d between the two slits must be of the order of a milimetre or so. The width a of each slit must be even smaller, of the order of 0.1 or 0.2 mm.

In our discussion of Young's experiment and the single-slit diffraction, we have assumed that the screen on which the fringes are formed is at a large distance. The two or more paths from the slits to the screen were treated as parallel. This situation also occurs when we place a converging lens after the slits and place the screen at the focus. Parallel paths from the slit are combined at a single point on the screen. *Note that the lens does not introduce any extra path differences in a parallel beam*. This arrangement is often used since it gives more intensity than placing the screen far away. If f is the focal length of the lens, then we can easily work out the size of the central bright maximum. In terms of angles, the separation of the central maximum from the first null of the diffraction pattern is λ/a. Hence, the size on the screen will be $f\lambda/a$.

10.6.2 Seeing the single slit diffraction pattern

It is surprisingly easy to see the single-slit diffraction pattern for oneself. The equipment needed can be found in most homes — two razor blades and one clear glass electric bulb preferably with a straight filament. One has to hold the two blades so that the edges are parallel and have a narrow slit in between. This is easily done with the thumb and forefingers (Fig. 10.18).

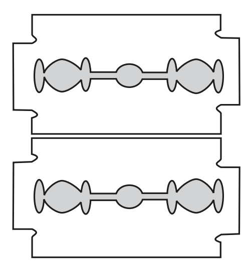


FIGURE 10.18 Holding two blades to form a single slit. A bulb filament viewed through this shows clear diffraction bands.

Keep the slit parallel to the filament, right in front of the eye. Use spectacles if you normally do. With slight adjustment of the width of the slit and the parallelism of the edges, the pattern should be seen with its bright and dark bands. Since the position of all the bands (except the central one) depends on wavelength, they will show some colours. Using a filter for red or blue will make the fringes clearer. With both filters available, the wider fringes for red compared to blue can be seen.

In this experiment, the filament plays the role of the first slit S in Fig. 10.16. The lens of the eye focuses the pattern on the screen (the retina of the eye).

With some effort, one can cut a double slit in an aluminium foil with a blade. The bulb filament can be viewed as before to repeat Young's experiment. In daytime, there is another suitable bright source subtending a small angle at the eye. This is the reflection of the Sun in any shiny convex surface (e.g., a cycle bell). Do not try direct sunlight – it can damage the eye and will not give fringes anyway as the Sun subtends an angle of $(1/2)°$.

In interference and diffraction, light energy is redistributed. If it reduces in one region, producing a dark fringe, it increases in another region, producing a bright fringe. There is no gain or loss of energy, which is consistent with the principle of conservation of energy.

10.6.3 Resolving power of optical instruments

In Chapter 9 we had discussed about telescopes. The angular resolution of the telescope is determined by the objective of the telescope. The stars which are not resolved in the image produced by the objective cannot be resolved by any further magnification produced by the eyepiece. The primary purpose of the eyepiece is to provide magnification of the image produced by the objective.

Consider a parallel beam of light falling on a convex lens. If the lens is well corrected for aberrations, then geometrical optics tells us that the beam will get focused to a point. However, because of diffraction, the beam instead of getting focused to a point gets focused to a spot of finite area. In this case the effects due to diffraction can be taken into account by considering a plane wave incident on a circular aperture followed by a convex lens (Fig. 10.19). The analysis of the corresponding diffraction pattern is quite involved; however, in principle, it is similar to the analysis carried out to obtain the single-slit diffraction pattern. Taking into account the effects due to diffraction, the pattern on the focal plane would consist of a central bright region surrounded by concentric dark and bright rings (Fig. 10.19). A detailed analysis shows that the radius of the central bright region is approximately given by

$$r_0 \approx \frac{1.22\,\lambda f}{2a} = \frac{0.61\lambda f}{a} \tag{10.24}$$

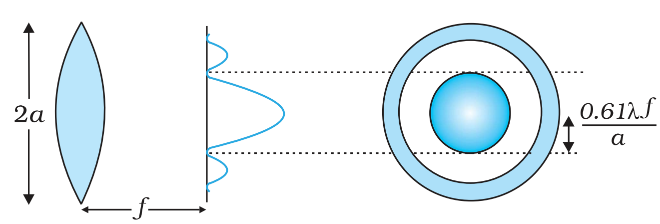


FIGURE 10.19 A parallel beam of light is incident on a convex lens. Because of diffraction effects, the beam gets focused to a spot of radius ≈ $0.61\ \lambda f/a$.

where f is the focal length of the lens and $2a$ is the diameter of the circular aperture or the diameter of the lens, whichever is smaller. Typically if

$$\lambda \approx 0.5\ \mu\text{m},\ f \approx 20\ \text{cm and } a \approx 5\ \text{cm}$$

we have

$$r_0 \approx 1.2\ \mu\text{m}$$

Although the size of the spot is very small, it plays an important role in determining the limit of resolution of optical instruments like a telescope or a microscope. For the two stars to be just resolved

$$f\Delta\theta \approx r_0 \approx \frac{0.61\lambda f}{a}$$

implying

$$\Delta\theta \approx \frac{0.61\lambda}{a} \qquad (10.25)$$

Thus $\Delta\theta$ will be small if the diameter of the objective is large. This implies that the telescope will have better resolving power if a is large. It is for this reason that for better resolution, a telescope must have a large diameter objective.

Example 10.6 Assume that light of wavelength 6000Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 100 inch?

Solution A 100 inch telescope implies that $2a$ = 100 inch = 254 cm. Thus if,

$$\lambda \approx 6000\text{Å} = 6\times10^{-5}\ \text{cm}$$

then

$$\Delta\theta \approx \frac{0.61\times6\times10^{-5}}{127} \approx 2.9\times10^{-7}\ \text{radians}$$

EXAMPLE 10.6

We can apply a similar argument to the objective lens of a microscope. In this case, the object is placed slightly beyond f, so that a real image is formed at a distance v [Fig. 10.20]. The magnification – ratio of image size to object size – is given by $m \approx v/f$. It can be seen from Fig. 10.20 that

$$D/f \approx 2\tan\beta \qquad (10.26)$$

where 2β is the angle subtended by the diameter of the objective lens at the focus of the microscope.

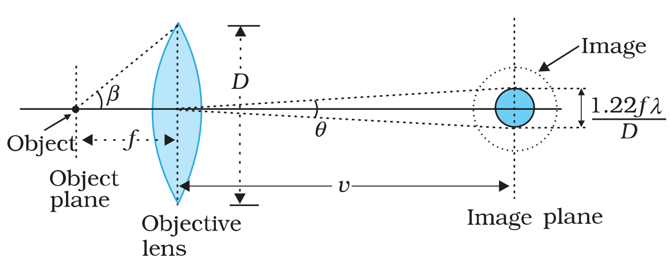
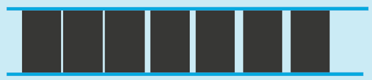


FIGURE 10.20 Real image formed by the objective lens of the microscope.

Determine the resolving power of your eye

You can estimate the resolving power of your eye with a simple experiment. Make black stripes of equal width separated by white stripes; see figure here. All the black stripes should be of equal width, while the width of the intermediate white stripes should increase as you go from the left to the right. For example, let all black stripes have a width of 5 mm. Let the width of the first two white stripes be 0.5 mm each, the next two white stripes be 1 mm each, the next two 1.5 mm each, etc. Paste this pattern on a wall in a room or laboratory, at the height of your eye.



Now watch the pattern, preferably with one eye. By moving away or closer to the wall, find the position where you can just see some two black stripes as separate stripes. All the black stripes to the left of this stripe would merge into one another and would not be distinguishable. On the other hand, the black stripes to the right of this would be more and more clearly visible. Note the width d of the white stripe which separates the two regions, and measure the distance D of the wall from your eye. Then d/D is the resolution of your eye.

You have watched specks of dust floating in air in a sunbeam entering through your window. Find the distance (of a speck) which you can clearly see and distinguish from a neighbouring speck. Knowing the resolution of your eye and the distance of the speck, estimate the size of the speck of dust.

When the separation between two points in a microscopic specimen is comparable to the wavelength λ of the light, the diffraction effects become important. The image of a point object will again be a diffraction pattern whose size in the image plane will be

$$v\,\theta = v\left(\frac{1.22\,\lambda}{D}\right) \tag{10.27}$$

Two objects whose images are closer than this distance will not be resolved, they will be seen as one. The corresponding minimum separation, d_{min}, in the object plane is given by

$$d_{min} = \left[v\left(\frac{1.22\,\lambda}{D}\right)\right]\bigg/m$$

$$= \frac{1.22\,\lambda}{D}\cdot\frac{v}{m}$$

$$= \frac{1.22\,f\,\lambda}{D} \tag{10.28}$$

Now, combining Eqs. (10.26) and (10.28), we get

$$d_{min} = \frac{1.22\,\lambda}{2\tan\beta}$$

$$\simeq \frac{1.22\,\lambda}{2\sin\beta} \qquad (10.29)$$

If the medium between the object and the objective lens is not air but a medium of refractive index *n*, Eq. (10.29) gets modified to

$$d_{\min} = \frac{1.22\,\lambda}{2\,n\sin\beta} \qquad (10.30)$$

The product $n\sin\beta$ is called the *numerical aperture* and is sometimes marked on the objective.

The resolving power of the microscope is given by the reciprocal of the minimum separation of two points seen as distinct. It can be seen from Eq. (10.30) that the resolving power can be increased by choosing a medium of higher refractive index. Usually an oil having a refractive index close to that of the objective glass is used. Such an arrangement is called an *'oil immersion objective'*. Notice that it is not possible to make $\sin\beta$ larger than unity. Thus, we see that the resolving power of a microscope is basically determined by the wavelength of the light used.

There is a likelihood of confusion between resolution and magnification, and similarly between the role of a telescope and a microscope to deal with these parameters. A telescope produces images of far objects nearer to our eye. Therefore objects which are not resolved at far distance, can be resolved by looking at them through a telescope. A microscope, on the other hand, magnifies objects (which are near to us) and produces their larger image. We may be looking at two stars or two satellites of a far-away planet, or we may be looking at different regions of a living cell. In this context, it is good to remember that a telescope resolves whereas a microscope magnifies.

10.6.4 The validity of ray optics

An aperture (i.e., slit or hole) of size *a* illuminated by a parallel beam sends diffracted light into an angle of approximately $\approx \lambda/a$. This is the angular size of the bright central maximum. In travelling a distance *z*, the diffracted beam therefore acquires a width $z\lambda/a$ due to diffraction. It is interesting to ask at what value of *z* the spreading due to diffraction becomes comparable to the size *a* of the aperture. We thus approximately equate $z\lambda/a$ with *a*. This gives the distance beyond which divergence of the beam of width *a* becomes significant. Therefore,

$$z \simeq \frac{a^2}{\lambda} \qquad (10.31)$$

We define a quantity z_F called the *Fresnel distance* by the following equation

$$z_F \simeq a^2/\lambda$$

Equation (10.31) shows that for distances much smaller than z_F, the spreading due to diffraction is smaller compared to the size of the beam. It becomes comparable when the distance is approximately z_F. For distances much greater than z_F, the spreading due to diffraction