Figure 9.27(b) explains the formation of primary rainbow. We see that red light from drop 1 and violet light from drop 2 reach the observers eye. The violet from drop 1 and red light from drop 2 are directed at level above or below the observer. Thus the observer sees a rainbow with red colour on the top and violet on the bottom. Thus, the primary rainbow is a result of three-step process, that is, refraction, reflection and refraction.

When light rays undergoes *two* internal reflections inside a raindrop, instead of *one* as in the primary rainbow, a secondary rainbow is formed as shown in Fig. 9.27(c). It is due to four-step process. The intensity of light is reduced at the second reflection and hence the secondary rainbow is fainter than the primary rainbow. Further, the order of the colours is reversed in it as is clear from Fig. 9.27(c).

9.8.2 Scattering of light

As sunlight travels through the earth's atmosphere, it gets *scattered* (changes its direction) by the atmospheric particles. Light of shorter wavelengths is scattered much more than light of longer wavelengths. (The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as Rayleigh scattering). Hence, the bluish colour predominates in a clear sky, since blue has a shorter wavelength than red and is scattered much more strongly. In fact, violet gets scattered even more than blue, having a shorter wavelength. But since our eyes are more sensitive to blue than violet, we see the sky blue.

Large particles like dust and water droplets present in the atmosphere behave differently. The relevant quantity here is the relative size of the wavelength of light λ, and the scatterer (of typical size, say, a). For $a \ll \lambda$, one has Rayleigh scattering which is proportional to $(1/\lambda)^4$. For $a \gg \lambda$, i.e., large scattering objects (for example, raindrops, large dust or ice particles) this is not true; all wavelengths are scattered nearly equally. Thus, clouds which have droplets of water with $a \gg \lambda$ are generally white.

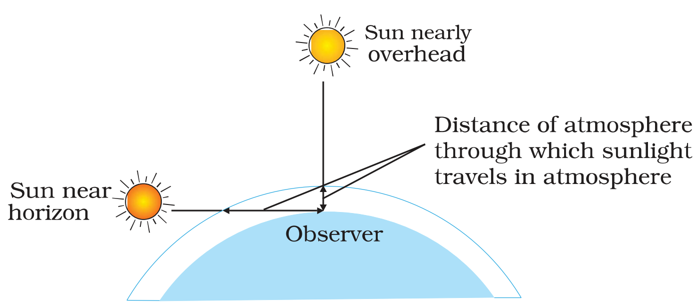

FIGURE 9.28 Sunlight travels through a longer distance in the atmosphere at sunset and sunrise.

At sunset or sunrise, the sun's rays have to pass through a larger distance in the atmosphere (Fig. 9.28). Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes, therefore, the sun looks reddish. This explains the reddish appearance of the sun and full moon near the horizon.

9.9 Optical Instruments

A number of optical devices and instruments have been designed utilising reflecting and refracting properties of mirrors, lenses and prisms. Periscope, kaleidoscope, binoculars, telescopes, microscopes are some examples of optical devices and instruments that are in common use. Our eye is, of course, one of the most important optical device the nature has endowed us with. Starting with the eye, we then go on to describe the principles of working of the microscope and the telescope.

9.9.1 The eye

Figure 9.29 (a) shows the eye. Light enters the eye through a curved front surface, the cornea. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles. For example, when the muscle is relaxed, the focal length is about 2.5 cm and objects at infinity are in sharp focus on the retina. When the object is brought closer to the eye, in order to maintain the same image-lens distance ($\cong$ 2.5 cm), the focal length of the eye lens becomes shorter by the action of the ciliary muscles. This property of the eye is called *accommodation*. If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred. The closest distance for which the lens can focus light on the retina is called the *least distance of distinct vision*, or the *near point*. The standard value for normal vision is taken as 25 cm. (Often the near point is given the symbol D.) This distance increases with age, because of the decreasing effectiveness of the ciliary muscle and the loss of flexibility of the lens. The near point may be as close as about 7 to 8 cm in a child ten years of age, and may increase to as much as 200 cm at 60 years of age. Thus, if an elderly person tries to read a book at about 25 cm from the eye, the image appears blurred. This condition (defect of the eye) is called *presbyopia*. It is corrected by using a converging lens for reading.

Thus, our eyes are marvellous organs that have the capability to interpret incoming electromagnetic waves as images through a complex process. These are our greatest assets and we must take proper care to protect them. Imagine the world without a pair of functional eyes. Yet many amongst us bravely face this challenge by effectively overcoming their limitations to lead a normal life. They deserve our appreciation for their courage and conviction.

In spite of all precautions and proactive action, our eyes may develop some defects due to various reasons. We shall restrict our discussion to some common optical defects of the eye. For example, the light from a distant object arriving at the eye-lens may get converged at a point in front of the retina. This type of defect is called *nearsightedness* or *myopia*. This means that the eye is producing too much convergence in the incident beam. To compensate this, we interpose a concave lens between the eye and the object, with the diverging effect desired to get the image focussed on the retina [Fig. 9.29(b)].

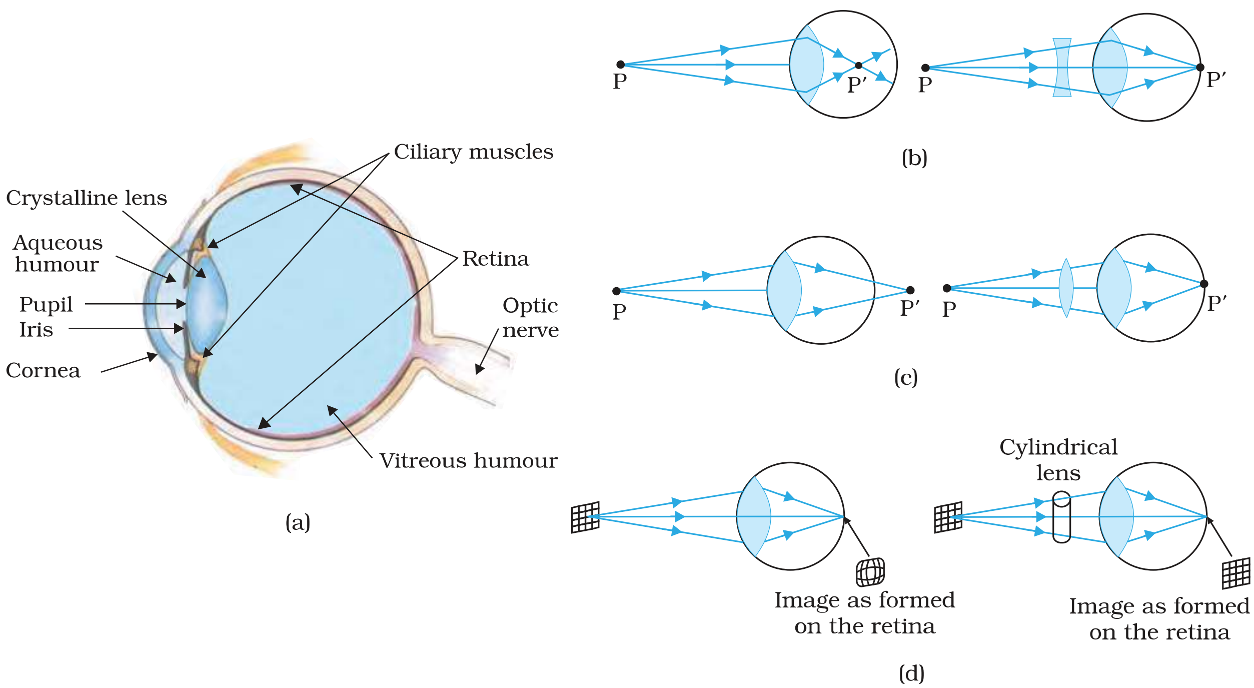

FIGURE 9.29 (a) The structure of the eye; (b) shortsighted or myopic eye and its correction; (c) farsighted or hypermetropic eye and its correction; and (d) astigmatic eye and its correction.

Similarly, if the eye-lens focusses the incoming light at a point behind the retina, a convergent lens is needed to compensate for the defect in vision. This defect is called *farsightedness* or *hypermetropia* [Fig. 9.29(c)].

Another common defect of vision is called *astigmatism.* This occurs when the cornea is not spherical in shape. For example, the cornea could have a larger curvature in the vertical plane than in the horizontal plane or vice-versa. If a person with such a defect in eye-lens looks at a wire mesh or a grid of lines, focussing in either the vertical or the horizontal plane may not be as sharp as in the other plane. Astigmatism results in lines in one direction being well focussed while those in a perpendicular direction may appear distorted [Fig. 9.29(d)]. Astigmatism can be corrected by using a cylindrical lens of desired radius of curvature with an appropriately directed axis. This defect can occur along with myopia or hypermetropia.

Example 9.10 What focal length should the reading spectacles have for a person for whom the least distance of distinct vision is 50 cm?

Solution The distance of normal vision is 25 cm. So if a book is at $u = -25$ cm, its image should be formed at $v = -50$ cm. Therefore, the desired focal length is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

or $\frac{1}{f} = \frac{1}{-50} - \frac{1}{-25} = \frac{1}{50}$

or $f = +50$ cm (convex lens).

EXAMPLE 9.10

Example 9.11

(a) The far point of a myopic person is 80 cm in front of the eye. What is the power of the lens required to enable him to see very distant objects clearly?

(b) In what way does the corrective lens help the above person? Does the lens magnify very distant objects? Explain carefully.

(c) The above person prefers to remove his spectacles while reading a book. Explain why?

Solution

(a) Solving as in the previous example, we find that the person should use a concave lens of focal length = – 80 cm, i.e., of power = – 1.25 dioptres.

(b) No. The concave lens, in fact, reduces the size of the object, but the angle subtended by the distant object at the eye is the same as the angle subtended by the image (at the far point) at the eye. The eye is able to see distant objects not because the corrective lens magnifies the object, but because it brings the object (i.e., it produces virtual image of the object) at the far point of the eye which then can be focussed by the eye-lens on the retina.

(c) The myopic person may have a normal near point, i.e., about 25 cm (or even less). In order to read a book with the spectacles, such a person must keep the book at a distance greater than 25 cm so that the image of the book by the concave lens is produced not closer than 25 cm. The angular size of the book (or its image) at the greater distance is evidently less than the angular size when the book is placed at 25 cm and no spectacles are needed. Hence, the person prefers to remove the spectacles while reading.

EXAMPLE 9.11

Example 9.12 (a) The near point of a hypermetropic person is 75 cm from the eye. What is the power of the lens required to enable the person to read clearly a book held at 25 cm from the eye? (b) In what way does the corrective lens help the above person? Does the lens magnify objects held near the eye? (c) The above person prefers to remove the spectacles while looking at the sky. Explain why?

Solution

(a) $u = -25$ cm, $v = -75$ cm

$1/f = 1/25 - 1/75$, i.e., $f = 37.5$ cm.

The corrective lens needs to have a converging power of +2.67 dioptres.

(b) The corrective lens produces a virtual image (at 75 cm) of an object at 25 cm. The angular size of this image is the same as that of the object. In this sense the lens does not magnify the object but merely brings the object to the near point of the hypermetric eye, which then gets focussed on the retina. However, the angular size is greater than that of the same object at the near point (75 cm) viewed without the spectacles.

(c) A hypermetropic eye may have normal far point i.e., it may have enough converging power to focus parallel rays from infinity on the retina of the shortened eyeball. Wearing spectacles of converging lenses (used for near vision) will amount to more converging power than needed for parallel rays. Hence the person prefers not to use the spectacles for far objects.

EXAMPLE 9.12

9.9.2 The microscope

A simple magnifier or microscope is a converging lens of small focal length (Fig. 9.30). In order to use such a lens as a microscope, the lens is held near the object, one focal length away or less, and the eye is positioned close to the lens on the other side. The idea is to get an erect, magnified and virtual image of the object at a distance so that it can be viewed comfortably, i.e., at 25 cm or more. If the object is at a distance *f*, the image is at infinity. However, if the object is at a distance slightly less than the focal length of the lens, the image is virtual and closer than infinity. Although the closest comfortable distance for viewing the image is when it is at the near point (distance $D \cong 25$ cm), it causes some strain on the eye. Therefore, the image formed at infinity is often considered most suitable for viewing by the relaxed eye. We show both cases, the first in Fig. 9.30(a), and the second in Fig. 9.30(b) and (c).

The linear magnification *m*, for the image formed at the near point *D*, by a simple microscope can be obtained by using the relation

$$m = \frac{v}{u} = v\left(\frac{1}{v} - \frac{1}{f}\right) = \left(1 - \frac{v}{f}\right)$$

Now according to our sign convention, *v* is negative, and is equal in magnitude to *D*. Thus, the magnification is

$$m = \left(1 + \frac{D}{f}\right) \qquad (9.39)$$

Since *D* is about 25 cm, to have a magnification of six, one needs a convex lens of focal length, $f = 5$ cm.

Note that $m = h'/h$ where *h* is the size of the object and h' the size of the image. This is also the ratio of the angle subtended by the image to that subtended by the object, if placed at *D* for comfortable viewing. (Note that this is not the angle actually subtended by the object at the eye, which is h/u.) What a single-lens simple magnifier achieves is that it allows the object to be brought closer to the eye than *D*.

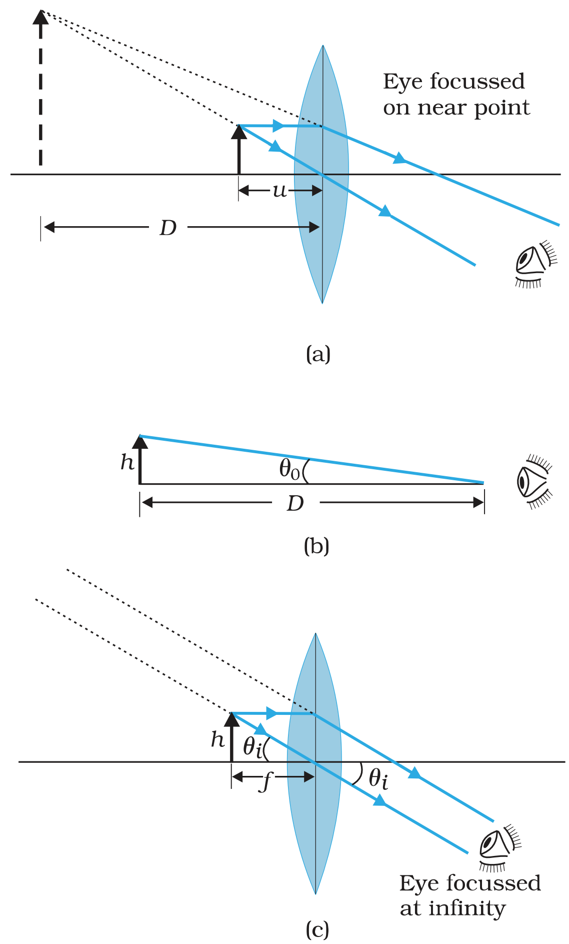

FIGURE 9.30 A simple microscope; (a) the magnifying lens is located such that the image is at the near point, (b) the angle subtended by the object, is the same as that at the near point, and (c) the object near the focal point of the lens; the image is far off but closer than infinity.

We will now find the magnification when the image is at infinity. In this case we will have to obtained the *angular* magnification. Suppose the object has a height *h*. The maximum angle it can subtend, and be clearly visible (without a lens), is when it is at the near point, i.e., a distance *D*. The angle subtended is then given by

$$\tan\,\theta_o = \left(\frac{h}{D}\right) \approx \theta_o \qquad (9.40)$$

We now find the angle subtended at the eye by the image when the object is at u. From the relations

$$\frac{h'}{h} = m = \frac{v}{u}$$

we have the angle subtended by the image

$\tan\theta_i = \frac{h'}{-v} = \frac{h}{-v}\cdot\frac{v}{u} = \frac{h}{-u} \approx \theta$. The angle subtended by the object, when it is at $u = -f$.

$$\theta_i = \left(\frac{h}{f}\right) \quad (9.41)$$

as is clear from Fig. 9.29(c). The angular magnification is, therefore

$$m = \left(\frac{\theta_i}{\theta_o}\right) = \frac{D}{f} \quad (9.42)$$

This is one less than the magnification when the image is at the near point, Eq. (9.39), but the viewing is more comfortable and the difference in magnification is usually small. In subsequent discussions of optical instruments (microscope and telescope) we shall assume the image to be at infinity.

A simple microscope has a limited maximum magnification (≤ 9) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a *compound microscope*. A schematic diagram of a compound microscope is shown in Fig. 9.31. The lens nearest the object, called the *objective*, forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the *eyepiece*, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object.

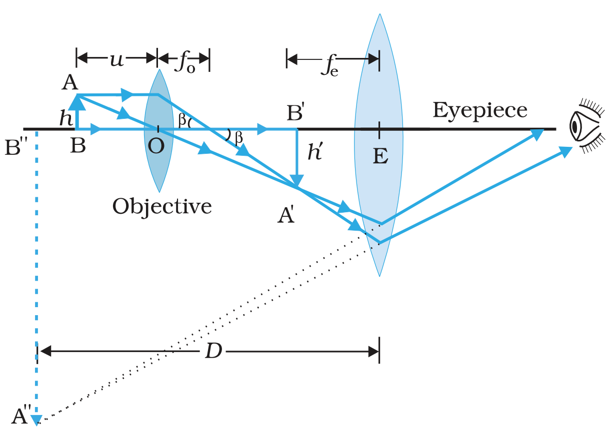

FIGURE 9.31 Ray diagram for the formation of image by a compound microscope.

We now obtain the magnification due to a compound microscope. The ray diagram of Fig. 9.31 shows that the (linear) magnification due to the objective, namely h'/h, equals

$$m_O = \frac{h'}{h} = \frac{L}{f_o} \quad (9.43)$$

where we have used the result

$$\tan\beta = \left(\frac{h}{f_o}\right) = \left(\frac{h'}{L}\right)$$

Here h' is the size of the first image, the object size being h and f_o being the focal length of the objective. The first image is formed near the focal point of the eyepiece. The distance L, i.e., the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_e) is called the tube length of the compound microscope.

As the first inverted image is near the focal point of the eyepiece, we use the result from the discussion above for the simple microscope to obtain the (angular) magnification m_e due to it [Eq. (9.39)], when the final image is formed at the near point, is

$$m_e = \left(1 + \frac{D}{f_e}\right) \qquad [9.44(a)]$$

When the final image is formed at infinity, the angular magnification due to the eyepiece [Eq. (9.42)] is

$$m_e = (D/f_e) \qquad [9.44(b)]$$

Thus, the total magnification [(according to Eq. (9.33)], when the image is formed at infinity, is

$$m = m_o m_e = \left(\frac{L}{f_o}\right) \left(\frac{D}{f_e}\right) \qquad (9.45)$$

Clearly, to achieve a large magnification of a *small* object (hence the name microscope), the objective and eyepiece should have small focal lengths. In practice, it is difficult to make the focal length much smaller than 1 cm. Also large lenses are required to make L large.

For example, with an objective with f_o = 1.0 cm, and an eyepiece with focal length f_e = 2.0 cm, and a tube length of 20 cm, the magnification is

$$m = m_o m_e = \left(\frac{L}{f_o}\right) \left(\frac{D}{f_e}\right)$$

$$= \frac{20}{1} \times \frac{25}{2} = 250$$

Various other factors such as illumination of the object, contribute to the quality and visibility of the image. In modern microscopes, multi-component lenses are used for both the objective and the eyepiece to improve image quality by minimising various optical aberrations (defects) in lenses.

9.9.3 Telescope

The telescope is used to provide angular magnification of distant objects (Fig. 9.32). It also has an objective and an eyepiece. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image. The magnifying power m is the ratio of the angle β subtended at the eye by the final image to the angle α which the object subtends at the lens or the eye. Hence

$$m \approx \frac{\beta}{\alpha} \approx \frac{h}{f_e} \cdot \frac{f_o}{h} = \frac{f_o}{f_e} \qquad (9.46)$$

In this case, the length of the telescope tube is $f_o + f_e$.

PHYSICS

The world's largest optical telescopes
http://astro.nineplanets.org/bigeyes.html

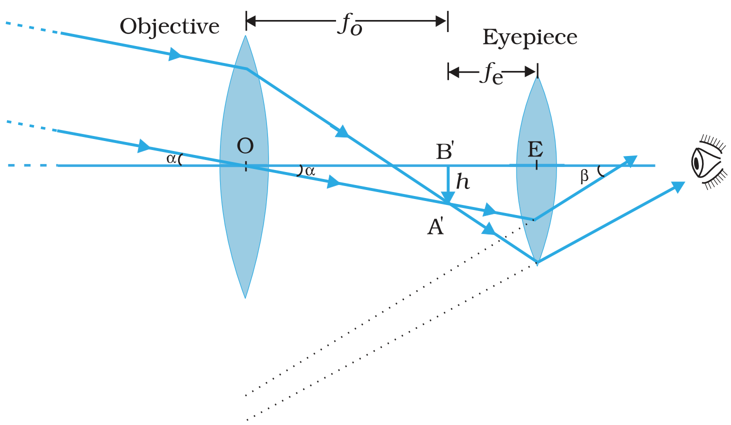

FIGURE 9.32 A refracting telescope.

Terrestrial telescopes have, in addition, a pair of inverting lenses to make the final image erect. Refracting telescopes can be used both for terrestrial and astronomical observations. For example, consider a telescope whose objective has a focal length of 100 cm and the eyepiece a focal length of 1 cm. The magnifying power of this telescope is m = 100/1 = 100.

Let us consider a pair of stars of actual separation 1′ (one minute of arc). The stars appear as though they are separated by an angle of 100 × 1′ = 100′ =1.67°.

The main considerations with an astronomical telescope are its light gathering power and its resolution or resolving power. The former clearly depends on the area of the objective. With larger diameters, fainter objects can be observed. The resolving power, or the ability to observe two objects distinctly, which are in very nearly the same direction, also depends on the diameter of the objective. So, the desirable aim in optical telescopes is to make them with objective of large diameter. The largest lens objective in use has a diameter of 40 inch (~1.02 m). It is at the Yerkes Observatory in Wisconsin, USA. Such big lenses tend to be very heavy and therefore, difficult to make and support by their edges. Further, it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions.

For these reasons, modern telescopes use a concave mirror rather than a lens for the objective. Telescopes with mirror objectives are called *reflecting* telescopes. They have several advantages. First, there is no chromatic aberration in a mirror. Second, if a parabolic reflecting surface is chosen, spherical aberration is also removed. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim. One obvious problem with a reflecting telescope is that the objective mirror focusses light inside the telescope tube. One must have an eyepiece and the observer right there, obstructing some light (depending on the size of the observer cage). This is what is done in the very large 200 inch (~5.08 m) diameters, Mt. Palomar telescope, California. The viewer sits near the focal point of the mirror, in a small cage. Another solution to the problem is to deflect the light being focussed by another mirror. One such arrangement using a convex secondary mirror to focus the incident light, which now passes through a hole in the objective primary mirror, is shown

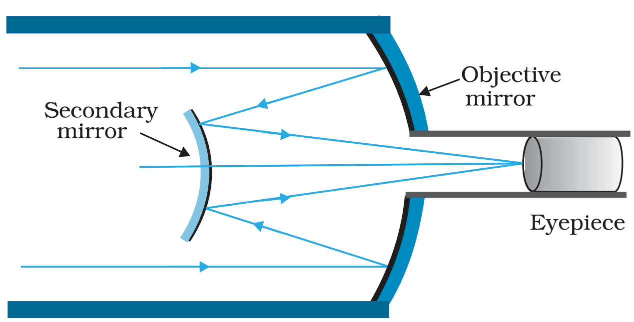

FIGURE 9.33 Schematic diagram of a reflecting telescope (Cassegrain).

in Fig. 9.33. This is known as a *Cassegrain* telescope, after its inventor. It has the advantages of a large focal length in a short telescope. The largest telescope in India is in Kavalur, Tamil Nadu. It is a 2.34 m diameter reflecting telescope (Cassegrain). It was ground, polished, set up, and is being used by the Indian Institute of Astrophysics, Bangalore. The largest reflecting telescopes in the world are the pair of Keck telescopes in Hawaii, USA, with a reflector of 10 metre in diameter.

SUMMARY

1. Reflection is governed by the equation $\angle i = \angle r'$ and refraction by the Snell's law, $\sin i/\sin r = n$, where the incident ray, reflected ray, refracted ray and normal lie in the same plane. Angles of incidence, reflection and refraction are i, r' and r, respectively.
2. The *critical angle of incidence* i_c for a ray incident from a denser to rarer medium, is that angle for which the angle of refraction is 90°. For $i > i_c$, total internal reflection occurs. Multiple internal reflections in diamond ($i_c \cong 24.4°$), totally reflecting prisms and mirage, are some examples of total internal reflection. Optical fibres consist of glass fibres coated with a thin layer of material of *lower* refractive index. Light incident at an angle at one end comes out at the other, after multiple internal reflections, even if the fibre is bent.
3. *Cartesian sign convention*: Distances measured in the same direction as the incident light are positive; those measured in the opposite direction are negative. All distances are measured from the pole/optic centre of the mirror/lens on the principal axis. The heights measured upwards above x-axis and normal to the principal axis of the mirror/lens are taken as positive. The heights measured downwards are taken as negative.
4. *Mirror equation*:

   $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

   where u and v are object and image distances, respectively and f is the focal length of the mirror. f is (approximately) half the radius of curvature R. f is negative for concave mirror; f is positive for a convex mirror.
5. For a prism of the angle A, of refractive index n_2 placed in a medium of refractive index n_1,

   $$n_{21} = \frac{n_2}{n_1} = \frac{\sin\left[\left(A + D_m\right)/2\right]}{\sin\left(A/2\right)}$$

   where D_m is the angle of minimum deviation.
6. *For refraction through a spherical interface* (from medium 1 to 2 of refractive index n_1 and n_2, respectively)

   $$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

   Thin lens formula

   $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$