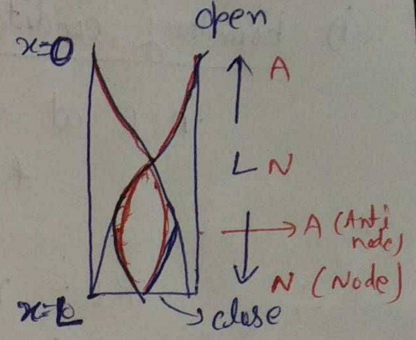
Formula sheet

① standing wave in pipe:-

a) closed organ pipe:-



pressure: $p(x,t) = A \sin kx \cos \omega t$

i) boundary condition:-

$p = 0$ at $x = 0$

$p \neq 0$ at $x = L$

$\therefore$ At $x = L$, pressure is max

$\therefore \quad kL = (2n+1)\frac{\pi}{2}$

$\because \quad k = \frac{2\pi}{\lambda}$

$\therefore \quad \frac{2\cancel{\pi}}{\lambda} \cdot L = (2n+1)\frac{\cancel{\pi}}{2}$

$$\boxed{\lambda_n = \frac{4L}{(4n+1)}}$$

(ii) $\therefore$ Allowed frequency:- $\nu_n = \frac{v}{\lambda_n}$

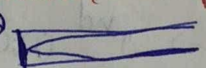
Note:- Only odd harmonics are present.

$$\boxed{\nu_n = (2n+1)\frac{v}{4L}}, \quad n = 0, 1, 2, \ldots$$

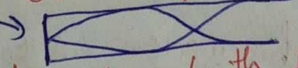
v = velocity of wave

L = length of pipe

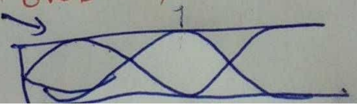
$\boxed{\nu_0 = \frac{v}{4L}}$ → fundamental frequency or 1st harmonics



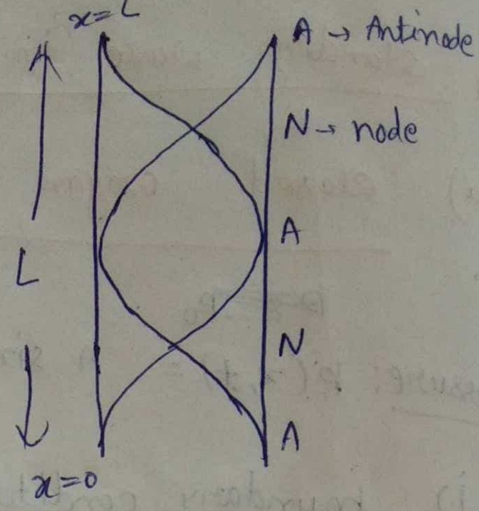
$\boxed{\nu_1 = \frac{3v}{4L}}$ → 1st overtone / 3rd harmonics



$\boxed{\nu_2 = \frac{5v}{4L}}$ → 2nd overtone / 5th harmonics



(5) Open organ pipe:-



pressure: $p = A \sin kx \cos \omega t$

i) boundary condition:-

$p = 0$ at $x = 0$
& $x = L$

$$kL = n\pi$$

$$\frac{2\pi}{\lambda} L = n\pi$$

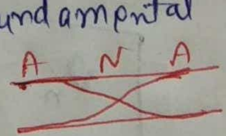
$$\boxed{\lambda_n = \frac{2L}{n}}$$

(ii) allowed frequencies:-

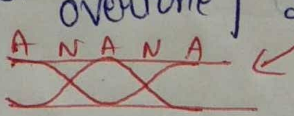
$$\nu_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$$

$$\boxed{\nu_n = \frac{nv}{2L}} \quad , n = 1, 2, 3, \text{—}$$

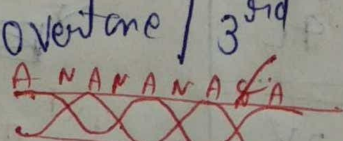
$\boxed{\nu_1 = \frac{v}{2L}}$ → fundamental frequency or 1st harmonics



$\boxed{\nu_2 = \frac{2v}{2L}}$ → 1st overtone / 2nd harmonics



$\boxed{\nu_3 = \frac{3v}{2L}}$ → 2nd overtone / 3rd harmonics



Note:- All harmonics are present.

(2) Beats:—

two waves of almost equal frequencies $\omega_1 \approx \omega_2$

e.g. $\omega_1 = 550\ Hz$ $\quad \Delta\omega << \omega_1$ or ω_2

$\omega_2 = 552\ Hz$

Let $p_1 = A \sin(\omega_1 t + kx)$

$p_2 = A \sin(\omega_2 t - kx)$

$\because$ $k = \frac{2\pi}{\lambda}$, $\omega = \frac{2\pi}{\nu}$, $\nu = \frac{v}{\lambda}$ → velocity; ν → frequency.

then $p = p_1 + p_2$

$= 2A \cos\Delta\omega\left(t - \frac{x}{v}\right) \sin\omega\left(t - \frac{x}{v}\right)$

$\Rightarrow \omega = \frac{(\omega_1 + \omega_2)}{2}$

$\boxed{\Delta\omega = \omega_1 - \omega_2}$ → beat frequency.

(3) Doppler Effect:—

Let speed of sound $= v$

initial frequency $= \nu$

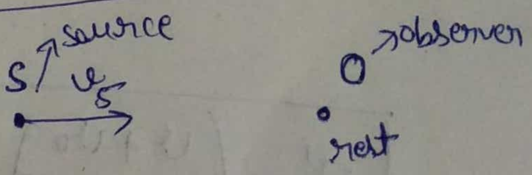
observed frequency $= \nu'$

Observer speed $= \nu_o$

Source speed $= \nu_s$

(i) when Source moving towards the ~~source:~~ observer:—

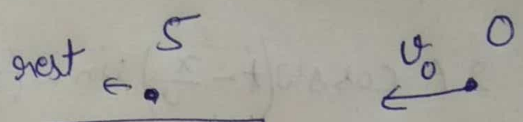
$$\boxed{\nu' = \left(\frac{v}{v - v_s}\right)\nu}$$



(ii) Source moving away from observer:—

$$\therefore \quad \boxed{\nu' = \left(\frac{v}{v+v_s}\right)\nu}$$

(iii) observer moving towards source:—

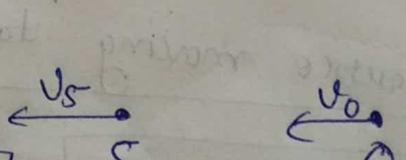


$$\boxed{\nu' = \left(\frac{v+v_o}{v}\right)\nu}$$

(iv) observer moving away from source:—

$$\boxed{\nu' = \left(\frac{v-v_o}{v}\right)\nu}$$

(v) Both source and observer moving:—



$$\boxed{\nu' = \left(\frac{v+v_o}{v+v_s}\right)\nu}$$

General formula:-

observed frequency $\rightarrow$ $$\boxed{\nu' = \left(\frac{v \pm v_o}{v \pm v_s}\right)\nu}$$ $\rightarrow$ original frequency

Case(i) : when Source is in rest.

i.e, $v_s = 0$

$$\nu' = \left(\frac{v \pm v_o}{v}\right)\nu$$

$+ \rightarrow$ when observer moves towards the source.

$- \rightarrow$ when observer moves away from the Source.

Case(ii):- when observer is in rest:-

i.e, $v_o = 0$

$$\nu' = \left(\frac{v}{v \pm v_s}\right)\nu$$

$+ \rightarrow$ when Source moves away from the source.

$- \rightarrow$ when source moves towards the source.