If on the other hand, the boundary point is not rigid but completely free to move (such as in the case of a string tied to a freely moving ring on a rod), the reflected pulse has the same phase and amplitude (assuming no energy dissipation) as the incident pulse. The net maximum displacement at the boundary is then twice the amplitude of each pulse. An example of non- rigid boundary is the open end of an organ pipe.

To summarise, a travelling wave or pulse suffers a phase change of π on reflection at a rigid boundary and no phase change on reflection at an open boundary. To put this mathematically, let the incident travelling wave be

$$y_2(x,t) = a\sin(kx - \omega t)$$

At a rigid boundary, the reflected wave is given by

$$y_r(x, t) = a \sin (kx - \omega t + \pi).$$
$$= - a \sin (kx - \omega t) \quad (15.35)$$

At an open boundary, the reflected wave is given by

$$y_r(x, t) = a \sin (kx - \omega t + 0).$$
$$= a \sin (kx - \omega t) \quad (15.36)$$

Clearly, at the rigid boundary, $y = y_2 + y_r = 0$ at all times.

15.6.1 Standing Waves and Normal Modes

We considered above reflection at one boundary. But there are familiar situations (a string fixed at either end or an air column in a pipe with either end closed) in which reflection takes place at two or more boundaries. In a string, for example, a wave travelling in one direction will get reflected at one end, which in turn will travel and get reflected from the other end. This will go on until there is a steady wave pattern set up on the string. Such wave patterns are called standing waves or stationary waves. To see this mathematically, consider a wave travelling along the positive direction of x-axis and a reflected wave of the same amplitude and wavelength in the negative direction of x-axis. From Eqs. (15.2) and (15.4), with $\phi = 0$, we get:

$$y_1(x, t) = a \sin (kx - \omega t)$$

$$y_2(x, t) = a \sin (kx + \omega t)$$

The resultant wave on the string is, according to the principle of superposition:

$$y (x, t) = y_1(x, t) + y_2(x, t)$$

$$= a [\sin (kx - \omega t) + \sin (kx + \omega t)]$$

Using the familiar trignometric identity Sin (A+B) + Sin (A–B) = 2 sin A cosB we get,

$$y (x, t) = 2a \sin kx \cos \omega t \quad (15.37)$$

Note the important difference in the wave pattern described by Eq. (15.37) from that described by Eq. (15.2) or Eq. (15.4). The terms kx and ωt appear separately, not in the combination $kx - \omega t$. The amplitude of this wave is $2a \sin kx$. Thus, in this wave pattern, the amplitude varies from point-to-point, but each element of the string oscillates with the same angular frequency ω or time period. There is no phase difference between oscillations of different elements of the wave. The string as a whole vibrates in phase with differing amplitudes at different points. The wave pattern is neither moving to the right nor to the left. Hence, they are called standing or stationary waves. The amplitude is fixed at a given location but, as remarked earlier, it is different at different locations. The points at which the amplitude is zero (i.e., where there is no motion at all) are **nodes**; the points at which the amplitude is the largest are called **antinodes.** Fig. 15.12 shows a stationary wave pattern resulting from superposition of two travelling waves in opposite directions.

The most significant feature of stationary waves is that the boundary conditions constrain the possible wavelengths or frequencies of vibration of the system. The system cannot oscillate with any arbitrary frequency (contrast this with a harmonic travelling wave), but is characterised by a set of natural frequencies or **normal modes** of oscillation. Let us determine these normal modes for a stretched string fixed at both ends.

First, from Eq. (15.37), the positions of nodes (where the amplitude is zero) are given by $\sin kx = 0$.

which implies

$$kx = n\pi; \quad n = 0, 1, 2, 3, ...$$

Since, $k = 2\pi/\lambda$, we get

$$x = \frac{n\lambda}{2} \; ; n = 0, 1, 2, 3, ... \quad (15.38)$$

Clearly, the distance between any two successive nodes is $\frac{\lambda}{2}$. In the same way, the

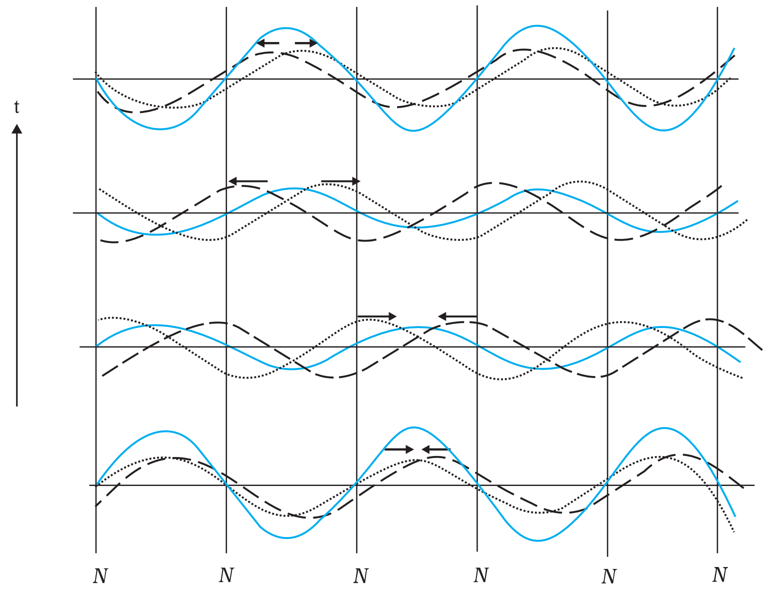


Fig. 15.12 *Stationary waves arising from superposition of two harmonic waves travelling in opposite directions. Note that the positions of zero displacement (nodes) remain fixed at all times.*

positions of antinodes (where the amplitude is the largest) are given by the largest value of sin kx:

$|\sin kx| = 1$

which implies

$kx = (n + ½)\,\pi\,;\ n = 0, 1, 2, 3, ...$

With $k = 2\pi/\lambda$, we get

$$x = (n + ½)\frac{\lambda}{2}\ ;\ n = 0, 1, 2, 3, ... \qquad (15.39)$$

Again the distance between any two consecutive antinodes is $\frac{\lambda}{2}$. Eq. (15.38) can be applied to the case of a stretched string of length L fixed at both ends. Taking one end to be at $x = 0$, the boundary conditions are that $x = 0$ and $x = L$ are positions of nodes. The $x = 0$ condition is already satisfied. The $x = L$ node condition requires that the length L is related to λ by

$$L = n\,\frac{\lambda}{2};\quad n = 1, 2, 3, ... \qquad (15.40)$$

Thus, the possible wavelengths of stationary waves are constrained by the relation

$$\lambda = \frac{2L}{n};\quad n = 1, 2, 3, ... \qquad (15.41)$$

with corresponding frequencies

$$\nu = \frac{nv}{2L},\ \text{for}\ n = 1, 2, 3, \qquad (15.42)$$

We have thus obtained the natural frequencies - the normal modes of oscillation of the system. The lowest possible natural frequency of a system is called its **fundamental mode** or the **first harmonic**. For the stretched string fixed at either end it is given by $\nu = \frac{v}{2L}$, corresponding to $n = 1$ of Eq. (15.42). Here v is the speed of wave determined by the properties of the medium. The $n = 2$ frequency is called the second harmonic; $n = 3$ is the third harmonic and so on. We can label the various harmonics by the symbol ν_n (n = 1, 2, ...).

Fig. 15.13 shows the first six harmonics of a stretched string fixed at either end. A string need not vibrate in one of these modes only. Generally, the vibration of a string will be a superposition of different modes; some modes may be more strongly excited and some less. Musical instruments like sitar or violin are based on this principle. Where the string is plucked or bowed, determines which modes are more prominent than others.

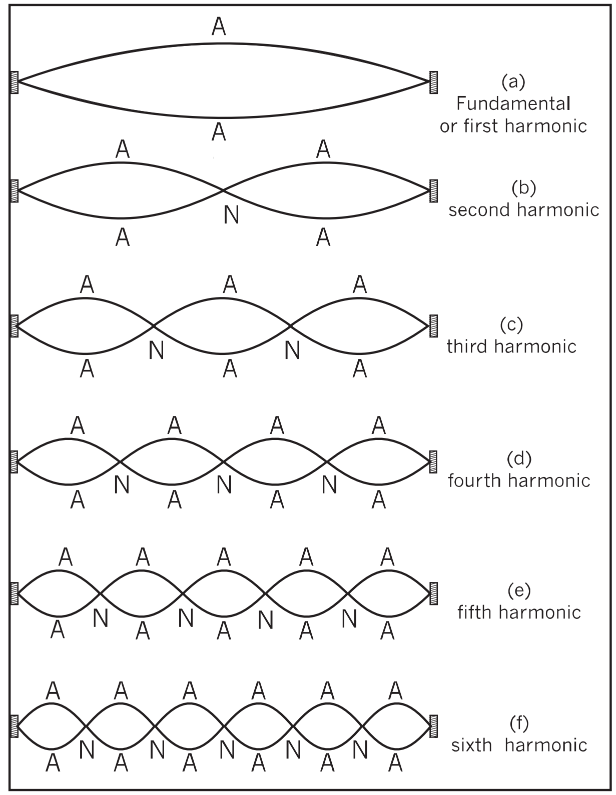


Fig. 15.13 *The first six harmonics of vibrations of a stretched string fixed at both ends.*

Let us next consider normal modes of oscillation of an air column with one end closed and the other open. A glass tube partially filled with water illustrates this system. The end in contact with water is a node, while the open end is an antinode. At the node the pressure changes are the largest, while the displacement is minimum (zero). At the open end - the antinode, it is just the other way - least pressure change and maximum amplitude of displacement. Taking the end in contact with water to be $x = 0$, the node condition (Eq. 15.38) is already satisfied. If the other end $x = L$ is an antinode, Eq. (15.39) gives

$$L = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \text{ for } n = 0, 1, 2, 3, \ldots$$

The possible wavelengths are then restricted by the relation :

$$\lambda = \frac{2L}{(n + 1/2)}, \text{ for } n = 0, 1, 2, 3, \ldots \quad (15.43)$$

The normal modes – the natural frequencies – of the system are

$$\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}; \; n = 0, 1, 2, 3, \ldots \quad (15.44)$$

The fundamental frequency corresponds to $n = 0$, and is given by $\frac{v}{4L}$. The higher frequencies are **odd harmonics**, i.e., odd multiples of the fundamental frequency : $3\frac{v}{4L}$, $5\frac{v}{4L}$, etc. Fig. 15.14 shows the first six odd harmonics of air column with one end closed and the other open. For a pipe open at both ends, each end is an antinode. It is then easily seen that an open air column at both ends generates all harmonics (See Fig. 15.15).

The systems above, strings and air columns, can also undergo forced oscillations (Chapter 14). If the external frequency is close to one of the natural frequencies, the system shows **resonance**.

Normal modes of a circular membrane rigidly clamped to the circumference as in a tabla are determined by the boundary condition that no point on the circumference of the membrane vibrates. Estimation of the frequencies of normal modes of this system is more complex. This problem involves wave propagation in two dimensions. However, the underlying physics is the same.

Example 15.5 A pipe, 30.0 cm long, is open at both ends. Which harmonic mode of the pipe resonates a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed ? Take the speed of sound in air as 330 m s^{-1}.

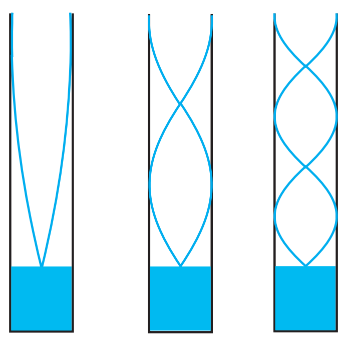
Answer The first harmonic frequency is given by

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \quad \text{(open pipe)}$$

where L is the length of the pipe. The frequency of its nth harmonic is:

$$\nu_n = \frac{nv}{2L}, \text{ for } n = 1, 2, 3, \ldots \text{ (open pipe)}$$

First few modes of an open pipe are shown in Fig. 15.15.



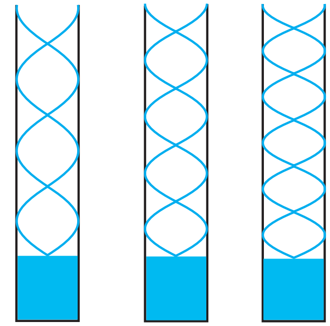


Fig. 15.14 *Normal modes of an air column open at one end and closed at the other end. Only the odd harmonics are seen to be possible.*

For L = 30.0 cm, v = 330 m s^{-1},

$$\nu_n = \frac{n\ 330\ (\text{m s}^{-1})}{0.6\ (\text{m})} = 550\ n\ \text{s}^{-1}$$

Clearly, a source of frequency 1.1 kHz will resonate at v_2, i.e. the **second harmonic**.

Now if one end of the pipe is closed (Fig. 15.15), it follows from Eq. (14.50) that the fundamental frequency is

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \quad \text{(pipe closed at one end)}$$

and only the odd numbered harmonics are present :

$$\nu_3 = \frac{3v}{4L},\ \nu_5 = \frac{5v}{4L}, \text{ and so on.}$$

For L = 30 cm and v = 330 m s^{-1}, the fundamental frequency of the pipe closed at one end is 275 Hz and the source frequency corresponds to its fourth harmonic. Since this harmonic is not a possible mode, no resonance will be observed with the source, the moment one end is closed.

15.7 BEATS

'Beats' is an interesting phenomenon arising from interference of waves. When two harmonic sound waves of close (but not equal) frequencies

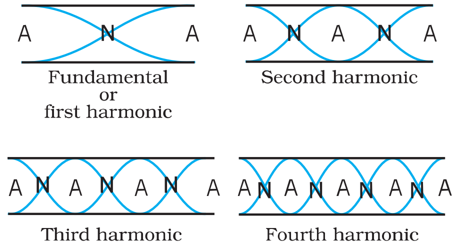


Fig. 15.15 *Standing waves in an open pipe, first four harmonics are depicted.*

are heard at the same time, we hear a sound of similar frequency (the average of two close frequencies), but we hear something else also. We hear audibly distinct waxing and waning of the intensity of the sound, with a frequency equal to the difference in the two close frequencies. Artists use this phenomenon often while tuning their instruments with each other. They go on tuning until their sensitive ears do not detect any beats.

To see this mathematically, let us consider two harmonic sound waves of nearly equal angular frequency ω_1 and ω_2 and fix the location to be x = 0 for convenience. Eq. (15.2) with a suitable choice of phase ($\phi = \pi/2$ for each) and, assuming equal amplitudes, gives

$$s_1 = a \cos \omega_1 t \quad \text{and} \quad s_2 = a \cos \omega_2 t \qquad (15.45)$$

Here we have replaced the symbol y by *s*, since we are referring to longitudinal not transverse displacement. Let ω_1 be the (slightly) greater of the two frequencies. The resultant displacement is, by the principle of superposition,

$$s = s_1 + s_2 = a(\cos \omega_1 t + \cos \omega_2 t)$$

Using the familiar trignometric identity for cos A + cosB, we get

$$= 2\,a \cos \frac{(\omega_1 - \omega_2)t}{2} \cos \frac{(\omega_1 + \omega_2)t}{2} \qquad (15.46)$$

which may be written as :

$$s = [2\,a \cos \omega_b t] \cos \omega_a t \qquad (15.47)$$

If $|\omega_1 - \omega_2| << \omega_1, \omega_2, \omega_a >> \omega_b$, th where

$$\omega_b = \frac{(\omega_1 - \omega_2)}{2} \quad \text{and} \quad \omega_a = \frac{(\omega_1 + \omega_2)}{2}$$

Now if we assume $|\omega_1 - \omega_2| << \omega_1$, which means $\omega_a >> \omega_b$, we can interpret Eq. (15.47) as follows. The resultant wave is oscillating with the average angular frequency ω_a; however its amplitude is **not** constant in time, unlike a pure harmonic wave. The amplitude is the largest when the term *cos* $\omega_b t$ takes its limit +1 or –1. In other words, the intensity of the resultant wave waxes and wanes with a frequency which is $2\omega_b = \omega_1 -$

Musical Pillars

Temples often have some pillars portraying human figures playing musical instruments, but seldom do these pillars themselves produce music. At the Nellaiappar temple in Tamil Nadu, gentle taps on a cluster of pillars carved out of a single piece of rock produce the basic notes of Indian classical music, viz. Sa, Re, Ga, Ma, Pa, Dha, Ni, Sa. Vibrations of these pillars depend on elasticity of the stone used, its density and shape.

Musical pillars are categorised into three types: The first is called the **Shruti Pillar**, as it can produce the basic notes — the "swaras". The second type is the **Gana Thoongal**, which generates the basic tunes that make up the "ragas". The third variety is the **Laya Thoongal** pillars that produce "taal" (beats) when tapped. The pillars at the Nellaiappar temple are a combination of the Shruti and Laya types.

Archaeologists date the Nelliappar temple to the 7th century and claim it was built by successive rulers of the Pandyan dynasty.

The musical pillars of Nelliappar and several other temples in southern India like those at Hampi (picture), Kanyakumari, and Thiruvananthapuram are unique to the country and have no parallel in any other part of the world.

ω_2. Since $\omega = 2\pi\nu$, the beat frequency ν_{beat}, is given by

$$\nu_{beat} = \nu_1 - \nu_2 \qquad (15.48)$$

Fig. 15.16 illustrates the phenomenon of beats for two harmonic waves of frequencies 11 Hz and 9 Hz. The amplitude of the resultant wave shows beats at a frequency of 2 Hz.

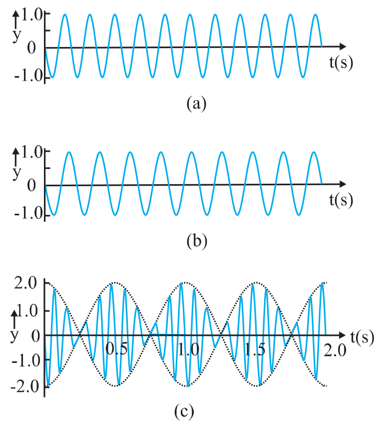


Fig. 15.16 *Superposition of two harmonic waves, one of frequency 11 Hz (a), and the other of frequency 9Hz (b), giving rise to beats of frequency 2 Hz, as shown in (c).*

Example 15.6 Two sitar strings A and B playing the note '*Dha*' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz ?

Answer Increase in the tension of a string increases its frequency. If the original frequency of B (ν_B) were greater than that of A (ν_A), further increase in ν_B should have resulted in an increase in the beat frequency. But the beat frequency is found to decrease. This shows that $\nu_B < \nu_A$. Since $\nu_A - \nu_B = 5$ Hz, and $\nu_A = 427$ Hz, we get $\nu_B = 422$ Hz.

15.8 DOPPLER EFFECT

It is an everyday experience that the pitch (or frequency) of the whistle of a fast moving train decreases as it recedes away. When we approach a stationary source of sound with high speed, the pitch of the sound heard appears to be higher than that of the source. As the observer recedes away from the source, the observed **pitch** (or frequency) becomes lower than that of the source. This motion-related frequency change is called **Doppler effect**. The Austrian physicist Johann Christian Doppler first proposed the effect in 1842. Buys Ballot in Holland tested it experimentally in 1845. Doppler effect is a wave phenomenon, it holds not only for sound waves but also for electromagnetic waves. However, here we shall consider only sound waves.

We shall analyse changes in frequency under three different situations: (1) observer is

Reflection of sound in an open pipe

When a high pressure pulse of air travelling down an open pipe reaches the other end, its momentum drags the air out into the open, where pressure falls rapidly to the atmospheric pressure. As a result the air following after it in the tube is pushed out. The low pressure at the end of the tube draws air from further up the tube. The air gets drawn towards the open end forcing the low pressure region to move upwards. As a result a pulse of *high* pressure air travelling *down* the tube turns into a pulse of *low* pressure air travelling *up* the tube. We say a pressure wave has been reflected at the open end with a change in phase of 180°. Standing waves in an open pipe organ like the flute is a result of this phenomenon.

Compare this with what happens when a pulse of high pressure air arrives at a closed end: it collides and as a result pushes the air back in the opposite direction. Here, we say that the pressure wave is reflected, with no change in phase.

stationary but the source is moving, (2) observer is moving but the source is stationary, and (3) both the observer and the source are moving. The situations (1) and (2) differ from each other because of the absence or presence of relative motion between the observer and the medium. Most waves require a medium for their propagation; however, electromagnetic waves do not require any medium for propagation. If there is no medium present, the Doppler shifts are same irrespective of whether the source moves or the observer moves, since there is no way of distinction between the two situations.

15.8.1 Source Moving ; Observer Stationary

Let us choose the convention to take the direction from the observer to the source as the positive direction of velocity. Consider a source S moving with velocity υ_s and an observer who is stationary in a frame in which the medium is also at rest. Let the speed of a wave of angular frequency ω and period T_o, both measured by an observer at rest with respect to the medium, be υ. We assume that the observer has a detector that counts every time a wave crest reaches it. As shown in Fig. 15.17, at time $t = 0$ the source is at point S_1, located at a distance L from the observer, and emits a crest. This reaches the observer at time $t_1 = L/\upsilon$. At time $t = T_o$ the source has moved a distance $\upsilon_s T_o$ and is at point S_2, located at a distance $(L + \upsilon_s T_o)$ from the observer. At S_2, the source emits a second crest. This reaches the observer at

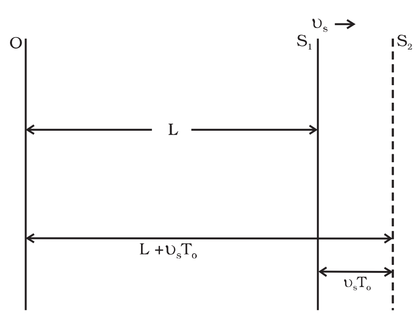


Fig. 15.17 *Doppler effect (change in frequency of wave) detected when the source is moving and the observer is at rest in the medium.*

$$t_2 = T_0 + \frac{(L + \upsilon_s T_0)}{\upsilon}$$

At time $n\,T_0$, the source emits its $(n+1)^{\text{th}}$ crest and this reaches the observer at time

$$t_{n+1} = n\,T_0 + \frac{(L + n\upsilon_s T_0)}{\upsilon}$$

Hence, in a time interval

$$\left[nT_0 + \frac{(L + n\upsilon_s T_0)}{\upsilon} - \frac{L}{\upsilon}\right]$$

the observer's detector counts n crests and the observer records the period of the wave as T given by

$$T = \left[nT_0 + \frac{(L + n\upsilon_s T_0)}{\upsilon} - \frac{L}{\upsilon}\right]/n$$

$$= T_0 + \frac{\upsilon_s T_0}{\upsilon}$$

$$= T_0\left(1 + \frac{\upsilon_s}{\upsilon}\right) \qquad (15.49)$$

Equation (15.49) may be rewritten in terms of the frequency ν_o that would be measured if the source and observer were stationary, and the frequency ν observed when the source is moving, as

$$\nu = \nu_0\left(1 + \frac{\upsilon_s}{\upsilon}\right)^{-1} \qquad (15.50)$$

If υ_s is small compared with the wave speed υ, taking binomial expansion to terms in first order in υ_s/υ and neglecting higher power, Eq. (15.50) may be approximated, giving

$$\nu = \nu_0\left(1 - \frac{\upsilon_s}{\upsilon}\right) \qquad (15.51)$$

For a source approaching the observer, we replace υ_s by $-\upsilon_s$ to get

$$\nu = \nu_0\left(1 + \frac{\upsilon_s}{\upsilon}\right) \qquad (15.52)$$

The observer thus measures a lower frequency when the source recedes from him than he does when it is at rest. He measures a higher frequency when the source approaches him.

15.8.2 Observer Moving; Source Stationary

Now to derive the Doppler shift when the observer is moving with velocity υ_o towards the source and the source is at rest, we have to proceed in a different manner. We work in the reference frame of the moving observer. In this reference frame the source and medium are approaching at speed v_o and the speed with which the wave approaches is $v_o + v$. Following a similar procedure as in the previous case, we find that the time interval between the arrival of the first and the $(n+1)$ th crests is

$$t_{n+1} - t_1 = n\ T_0 - \frac{n v_0 T_0}{v_0 + v}$$

The observer thus, measures the period of the wave to be

$$= T_0\left(1 - \frac{v_0}{v_0 + v}\right)$$

$$= T_0\left(1 + \frac{v_0}{v}\right)^{-1}$$

giving

$$\nu = \nu_0\left(1 + \frac{v_0}{v}\right) \qquad (15.53)$$

If $\frac{v_0}{v}$ is small, the Doppler shift is almost same whether it is the observer or the source moving since Eq. (15.53) and the approximate relation Eq. (15.51) are the same.

15.8.3 Both Source and Observer Moving

We will now derive a general expression for Doppler shift when both the source and the observer are moving. As before, let us take the direction from the observer to the source as the positive direction. Let the source and the observer be moving with velocities v_s and v_o respectively as shown in Fig.15.18. Suppose at time $t = 0$, the observer is at O_1 and the source is at S_1, O_1 being to the left of S_1. The source emits a wave of velocity v, of frequency ν and period T_0 all measured by an observer at rest with respect to the medium. Let L be the distance between O_1 and S_1 at $t = 0$, when the source emits the first crest. Now, since the observer is moving, the velocity of the wave relative to the observer is $v+v_0$. Therefore, the first crest reaches the observer at time $t_1 = L/(v+v_0)$. At time $t = T_0$, both the observer and the source have moved to their new positions O_2 and S_2 respectively. The new distance between the observer and the source, $O_2\ S_2$, would be $L+(v_s-v_0)\ T_0]$. At S_2, the source emits a second crest.

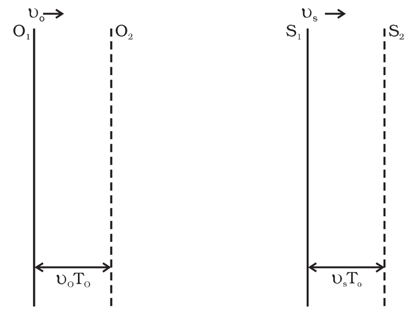


Fig. 15.18 *Doppler effect when both the source and observer are moving with different velocities.*

Application of Doppler effect

The change in frequency caused by a moving object due to Doppler effect is used to measure their velocities in diverse areas such as military, medical science, astrophysics, etc. It is also used by police to check over-speeding of vehicles.

A sound wave or electromagnetic wave of known frequency is sent towards a moving object. Some part of the wave is reflected from the object and its frequency is detected by the monitoring station. This change in frequency is called **Doppler shift**.

It is used at airports to guide aircraft, and in the military to detect enemy aircraft. Astrophysicists use it to measure the velocities of stars.

Doctors use it to study heart beats and blood flow in different parts of the body. Here they use ulltrasonic waves, and in common practice, it is called **sonography**. Ultrasonic waves enter the body of the person, some of them are reflected back, and give information about motion of blood and pulsation of heart valves, as well as pulsation of the heart of the foetus. In the case of heart, the picture generated is called **echocardiogram**.

This reaches the observer at time.

$$t_2 = T_o + [L + (v_s - v_o)T_o)]\ /(v + v_o)$$

At time nT_o the source emits its $(n+1)$ th crest and this reaches the observer at time

$$t_{n+1} = nT_o + [L + n\ (v_s - v_o)T_o)]\ /(v + v_o)$$

Hence, in a time interval $t_{n+1} - t_1$, i.e.,

$$nT_o + [L + n\ (v_s - v_o)T_o)]\ /(v + v_o) - L\ /(v + v_o),$$

the observer counts n crests and the observer records the period of the wave as equal to T given by

$$T = T_0\left(1+\frac{v_s - v_o}{v + v_0}\right) = T_0\left(\frac{v+v_s}{v+v_0}\right) \quad (15.54)$$

The frequency ν observed by the observer is given by

$$\nu = \nu_0\left(\frac{v+v_0}{v+v_s}\right) \quad (15.55)$$

Consider a passenger sitting in a train moving on a straight track. Suppose she hears a whistle sounded by the driver of the train. What frequency will she measure or hear? Here both the observer and the source are moving with the same velocity, so there will be no shift in frequency and the passenger will note the natural frequency. But an observer outside who is stationary with respect to the track will note a higher frequency if the train is approaching him and a lower frequency when it recedes from him.

Note that we have defined the direction from the observer to the source as the positive direction. Therefore, if the observer is moving towards the source, v_0 has a positive (numerical) value whereas if O is moving away from S, v_0 has a negative value. On the other hand, if S is moving away from O, v_s has a positive value whereas if it is moving towards O, v_s has a negative value. The sound emitted by the source travels in all directions. It is that part of sound coming towards the observer which the observer receives and detects. Therefore, the relative velocity of sound with respect to the observer is $v + v_0$ in all cases.

Example 15.7 A rocket is moving at a speed of 200 m s^{-1} towards a stationary target. While moving, it emits a wave of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. Calculate (1) the frequency of the sound as detected by the target and (2) the frequency of the echo as detected by the rocket.

Answer (1) The observer is at rest and the source is moving with a speed of 200 m s^{-1}. Since this is comparable with the velocity of sound, 330 m s^{-1}, we must use Eq. (15.50) and not the approximate Eq. (15.51). Since the source is approaching a stationary target, $v_o = 0$, and v_s must be replaced by $-v_s$. Thus, we have

$$\nu = \nu_0\left(1-\frac{v_s}{v}\right)^{-1}$$

$$\nu = 1000 \text{ Hz} \times [1 - 200 \text{ m s}^{-1}/330 \text{ m s}^{-1}]^{-1}$$

$$\simeq 2540 \text{ Hz}$$

(2) The target is now the source (because it is the source of echo) and the rocket's detector is now the detector or observer (because it detects echo). Thus, $v_s = 0$ and v_o has a positive value. The frequency of the sound emitted by the source (the target) is ν, the frequency intercepted by the target and not ν_o. Therefore, the frequency as registered by the rocket is

$$\nu' = \nu\left(\frac{v+v_0}{v}\right)$$

$$= 2540 \text{ Hz} \times \left(\frac{200 \text{ m s}^{-1} + 330 \text{ m s}^{-1}}{330 \text{ m s}^{-1}}\right)$$

$$\simeq 4080 \text{ Hz}$$

SUMMARY

1. *Mechanical waves* can exist in material media and are governed by Newton's Laws.
2. *Transverse waves* are waves in which the particles of the medium oscillate perpendicular to the direction of wave propagation.
3. *Longitudinal waves* are waves in which the particles of the medium oscillate along the direction of wave propagation.
4. *Progressive wave* is a wave that moves from one point of medium to another.
5. *The displacement* in a sinusoidal wave propagating in the positive x direction is given by

   $$y(x, t) = a \sin(kx - \omega t + \phi)$$

   where a *is the* amplitude of the wave, k is the angular wave number, ω *is the* angular frequency, $(kx - \omega t + \phi)$ *is the* phase, *and* ϕ *is the* phase constant or phase angle.
6. *Wavelength* λ of a progressive wave is the distance between two consecutive points of the same phase at a given time. In a stationary wave, it is twice the distance between two consecutive nodes or antinodes.
7. *Period* T of oscillation of a wave is defined as the time any element of the medium takes to move through one complete oscillation. It is related to the *angular frequency* ω through the relation

   $$T = \frac{2\pi}{\omega}$$

8. *Frequency* ν of a wave is defined as $1/T$ and is related to angular frequency by

   $$\nu = \frac{\omega}{2\pi}$$

9. *Speed* of a progressive wave is given by $\upsilon = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda\nu$
10. *The speed of a transverse wave* on a stretched string is set by the properties of the string. The speed on a string with tension T and linear mass density μ is

    $$\upsilon = \sqrt{\frac{T}{\mu}}$$

11. *Sound waves* are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed υ of sound wave in a fluid having *bulk modulus* B and density ρ is

    $$\upsilon = \sqrt{\frac{B}{\rho}}$$

    The speed of longitudinal waves in a metallic bar is

    $$\upsilon = \sqrt{\frac{Y}{\rho}}$$

    For gases, since $B = \gamma P$, the speed of sound is

    $$\upsilon = \sqrt{\frac{\gamma P}{\rho}}$$

12. When two or more waves traverse simultaneously in the same medium, the displacement of any element of the medium is the algebraic sum of the displacements due to each wave. This is known as the *principle of superposition* of waves

$$y = \sum_{i=1}^{n} f_i(x - vt)$$

13. Two sinusoidal waves on the same string exhibit *interference*, adding or cancelling according to the principle of superposition. If the two are travelling in the same direction and have the same amplitude a and frequency but differ in phase by a *phase constant* ϕ, the result is a single wave with the same frequency ω:

$$y(x, t) = \left[2a\cos\frac{1}{2}\phi\right] \sin\left(kx - \omega t + \frac{1}{2}\phi\right)$$

If $\phi = 0$ or an integral multiple of 2π, the waves are exactly in phase and the interference is constructive; if $\phi = \pi$, they are exactly out of phase and the interference is destructive.

14. A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal but the reflection at an open boundary takes place without any phase change.

For an incident wave

$$y_i(x, t) = a \sin(kx - \omega t)$$

the reflected wave at a rigid boundary is

$$y_r(x, t) = -a \sin(kx + \omega t)$$

For reflection at an open boundary

$$y_r(x, t) = a \sin(kx + \omega t)$$

15. The interference of two identical waves moving in opposite directions produces *standing waves*. For a string with fixed ends, the standing wave is given by

$$y(x, t) = [2a \sin kx] \cos \omega t$$

Standing waves are characterised by fixed locations of zero displacement called *nodes* and fixed locations of maximum displacements called *antinodes*. The separation between two consecutive nodes or antinodes is $\lambda/2$.

A stretched string of length L fixed at both the ends vibrates with frequencies given by

$$\nu = \frac{n v}{2L}, \qquad n = 1, 2, 3, \ldots$$

The set of frequencies given by the above relation are called the *normal modes* of oscillation of the system. The oscillation mode with lowest frequency is called the *fundamental mode* or the *first harmonic*. The *second harmonic* is the oscillation mode with $n = 2$ and so on.

A pipe of length L with one end closed and other end open (such as air columns) vibrates with frequencies given by

$$\nu = (n + ½)\frac{v}{2L}, \qquad n = 0, 1, 2, 3, \ldots$$

The set of frequencies represented by the above relation are the *normal modes* of oscillation of such a system. The lowest frequency given by $v/4L$ is the fundamental mode or the first harmonic.

16. A string of length L fixed at both ends or an air column closed at one end and open at the other end or open at both the ends, vibrates with certain frequencies called their normal modes. Each of these frequencies is a *resonant frequency* of the system.

17. *Beats* arise when two waves having slightly different frequencies, ν_1 and ν_2 and comparable amplitudes, are superposed. The beat frequency is

$$\nu_{beat} = \nu_1 \sim \nu_2$$

18. The *Doppler effect* is a change in the observed frequency of a wave when the source (S) or the observer (O) or both move(s) relative to the medium. For sound the observed frequency ν is given in terms of the source frequency ν_o by

$$\nu = \nu_o \left(\frac{\upsilon + \upsilon_0}{\upsilon + \upsilon_s} \right)$$

here υ is the speed of sound through the medium, υ_o is the velocity of observer relative to the medium, and υ_s is the source velocity relative to the medium. In using this formula, velocities in the direction OS should be treated as positive and those opposite to it should be taken to be negative.

Physical quantity	Symbol	Dimensions	Unit	Remarks
Wavelength	λ	[L]	m	Distance between two consecutive points with the same phase.
Propagation constant	k	[L^{-1}]	m^{-1}	$k = \frac{2\pi}{\lambda}$
Wave speed	υ	[LT^{-1}]	m s^{-1}	$\upsilon = \nu\lambda$
Beat frequency	ν_{beat}	[T^{-1}]	s^{-1}	Difference of two close frequencies of superposing waves.

POINTS TO PONDER

1. A wave is not motion of matter as a whole in a medium. A wind is different from the sound wave in air. The former involves motion of air from one place to the other. The latter involves compressions and rarefactions of layers of air.
2. In a wave, energy and *not the matter* is transferred from one point to the other.
3. In a mechanical wave, energy transfer takes place because of the coupling through elastic forces between neighbouring oscillating parts of the medium.
4. Transverse waves can propagate only in medium with shear modulus of elasticity, Longitudinal waves need bulk modulus of elasticity and are therefore, possible in all media, solids, liquids and gases.
5. In a harmonic progressive wave of a given frequency, all particles have the same amplitude but different phases at a given instant of time. In a stationary wave, all particles between two nodes have the same phase at a given instant but have different amplitudes.
6. Relative to an observer at rest in a medium the speed of a mechanical wave in that medium (υ) depends only on elastic and other properties (such as mass density) of the medium. It does not depend on the velocity of the source.
7. For an observer moving with velocity υ_o relative to the medium, the speed of a wave is obviously different from υ and is given by $\upsilon \pm \upsilon_o$.