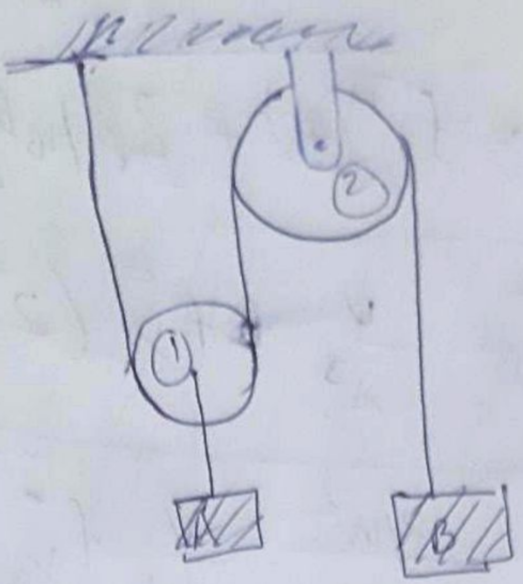
~~Two blocks~~ Problems on NLM

(8) (Easy)

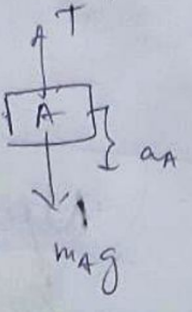


Two blocks A & B of masses m_A & m_B (massless) are shown with a given pulley system. Under what condition does block A has downward acceleration?

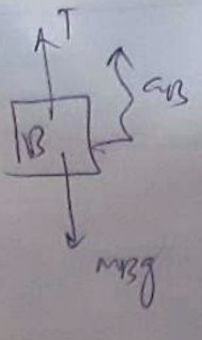
Ans:- F.B.D of blocks

Concepts Used
1. Constrained Pulley system.
Formulae Used
1. Newton's 2nd law of motion. $\vec{F}_{net} = M\vec{a}_{system}$
2. F.B.D

(A)



$m_A g - T' = m_A a_A$



$T - m_B g = m_B a_B$

m_A &

...en pulley

condition

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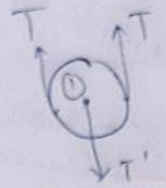
F.B.D of Pulley (1)



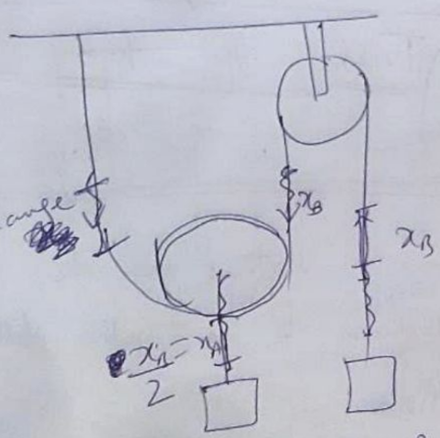
Since Pulley is massless

=> Applying Newton's 2nd Law of motion

=> $T' - 2T = 0$

=> $\boxed{T' = 2T}$

Now applying constraint relations :



If we displace Block B upwards by x_B then block A get shifted downwards by $x_B/2$.

=> $2x_A = x_B$

=> $2a_A = a_B$ [Differentiating twice]

Substituting the values

$$\Rightarrow m_A g - 2T = m_A a_A$$
$$T - m_B g = 2m_B a_A \quad \times 2$$

Adding

$$\Rightarrow 2T - 2m_B g = 4m_B a_A$$

$$\Rightarrow g(m_A - 2m_B) = (4m_B + m_A)\, a_A$$

$$\Rightarrow a_A = \frac{(m_A - 2m_B)g}{4m_B + m_A}$$

Now for acceleration for block A to be downwards $\Rightarrow a_A > 0$

$$\Rightarrow m_A - 2m_B > 0 \Rightarrow \boxed{m_A > 2m_B}$$

[Applicable Only in Pulley System]

Trick for finding Constraint Relation in Pulley Problems

We define virtual work for any system (usually block) (attached to a pulley system (usually string)) as follows:

$$\boxed{W_V = T_{sys} \cdot a_{sys} \cdot \cos\theta}$$

Hint: $\cos(0^\circ) = 1$, $\cos(180^\circ) = -1$

where T_{sys} = Tension in the string attached to the system

a_{sys} = acceleration of the system

Theorem :-

where

In the concept constraint

System - A

θ = Angle which the direction of tension makes with direction of acceleration of the system

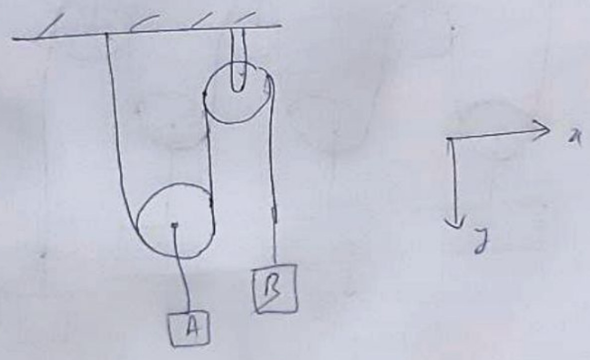
Theorem :- The sum total of virtual work of individual systems = 0 i.e.,

$$\boxed{\sum_i T_{sys_i} \cdot a_{sys_i} \cdot \cos\theta_i = 0}$$

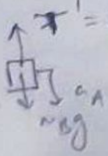
where i runs over all the systems.

Example :-

In the above problem we used the concept of virtual work to find the constraint relation.



System - A :-



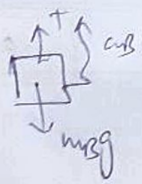
$T' = 2T$ (described in above example)

$T_{sys} = 2T$

$a_{sys} = a_A$

$\theta = 180°$

System - B :-



$T_{sys} = T$

$a_{sys} = a_B$

$\theta = 0°$

Applying the theorem :-

$$\sum_{i \in \{A,B\}} T_{sys_i} \cdot a_{sys_i} \cdot \cos\theta_i = 0$$ where A, B are systems.

$\Rightarrow \left(2T a_A \cos 180\right) + \left(T a_B \cos 0\right) = 0$

$\Rightarrow -2T a_A + T a_B = 0$

$\Rightarrow \boxed{2a_A = a_B}$ ⟶ Same as earlier.

4 (Easy)