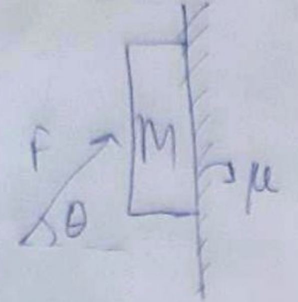
Problems on Laws of Motion :-

(Vinayak Dutta)

(Easy)

1)



A book of mass M is positioned against a vertical wall. The coefficient of friction between the book and wall is μ. You wish to keep the book from falling by pushing on it with a force F applied at an angle θ w.r.t the horizontal $\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$ as shown above.

(a) For a given θ, what is the minimum F required?

(b) For what θ is this minimum F the smallest? What is the corresponding minimum F?

(c) What is the limiting value of θ below which there does not exist an F that keeps the book up?

Sol:- **Concepts Used :-**

1. Balancing of forces in different directions by drawing Free body diagram (F.B.D.)

2. Concept of static friction.

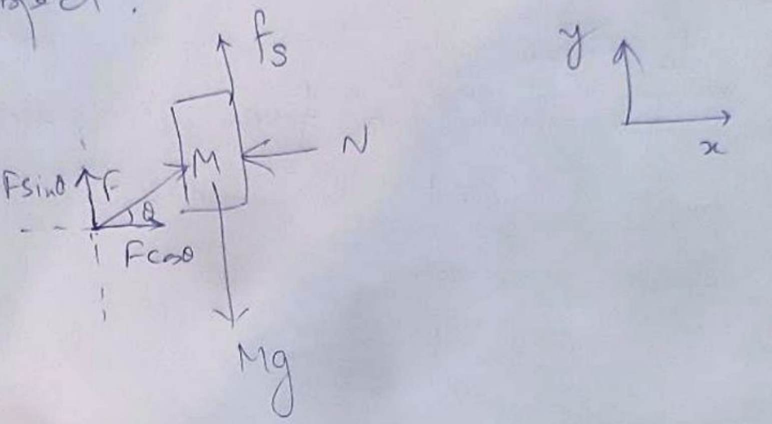
Formulae Used

1. Static friction $F_S \le \mu N$

where μ is coefficient of friction

N is the normal force.

(a) Let us draw F.B.D of the object.



Note that:-

f_s acts upwards since body has tendency to fall down.

Balancing the forces :

x - direction :-

$F\cos\theta = N$ —— (a)

y - direction :-

$F\sin\theta + f_s = Mg$

$\Rightarrow f_s = Mg - F\sin\theta \le \mu N = \mu F\cos\theta$ — (b)

from — (a)

From —(b)

$$Mg \leq F(\sin\theta + \mu\cos\theta)$$

So Assuming $\sin\theta + \mu\cos\theta > 0$

$$\Rightarrow \boxed{F \geq \frac{Mg}{\sin\theta + \mu\cos\theta}} \quad \text{—(c)}$$

For $\sin\theta + \mu\cos\theta \leq 0$

there exists no solution of F

(b) To minimise this lower bound, denominator must be maximised

$\Rightarrow (\sin\theta + \mu\cos\theta)' = 0$ [where $f' = \frac{df}{dx}$]

$\Rightarrow \cos\theta - \mu\sin\theta = 0$

$\Rightarrow \tan\theta = \frac{1}{\mu} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{1}{\mu}\right)}$

Putting this in (c)

$$\Rightarrow \boxed{F \geq \frac{mg}{\sqrt{1+\mu^2}}}$$

(c) As mentioned earlier for $\sin\theta + \mu\cos\theta \leq 0$.

$\Rightarrow$ Such an F doesn't exist.

So, $\sin\theta + \mu\cos\theta \leq 0$

$\Rightarrow \tan\theta \leq -\mu \Rightarrow \boxed{\theta \leq \tan^{-1}(-\mu)}$

Thus this is the limiting value of θ below which no such f exists.

(2)