Q. If $T_r = {}^nC_r (a^r - 1)$ find $\sum_{r=1}^{n} T_r$.

$\Rightarrow$ $T_1 = {}^nC_1 (a-1)$

$T_2 = {}^nC_2 (a^2-1)$

$T_3 = {}^nC_3 (a^3-1)$

$T_n = {}^nC_n (a^n-1)$

$\sum_{r=0}^{n} T_n \Rightarrow 2^{n+1}(a^n - n) - 1$

$\boxed{\sum_{r=1}^{n} T_r \Rightarrow 2^n (a^n - 1) - 1}$

$\Rightarrow \sum_{r=1}^{n} {}^nC_r (a^r - 1)$

$\Rightarrow \sum_{r=1}^{n} ({}^nC_r a^r - {}^nC_r)$

Teacher's Signature............

$$\Rightarrow \sum_{r=1}^{n} {}^{n}C_r a^r - \sum_{r=1}^{n} {}^{n}C_r$$

$$\Rightarrow \left(\sum_{r=0}^{n} {}^{n}C_r a^r - {}^{n}C_0 a^0\right) - \left(\sum_{r=0}^{n} {}^{n}C_r - {}^{n}C_0\right)$$

$$\Rightarrow \left((1+a)^n - 1\right) - \left(2^n - 1\right)$$

$$\Rightarrow (a+1)^n - \cancel{1} - 2^n + \cancel{1}$$

$$\Rightarrow (a+1)^n - 2^n$$

Type :- II

$$\boxed{{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r}$$

Q. ${}^{45}C_5 + {}^{45}C_4 + {}^{46}C_4 + {}^{47}C_4 + \cdots {}^{59}C_4$

$\Rightarrow {}^{45}C_5 + {}^{46}C_4 + {}^{47}C_4 + \cdots$

$+ {}^{59}C_4$

2021.06.11 17

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$\Rightarrow \quad {}^{47}C_5 + {}^{47}C_4 + \cdots + {}^{59}C_4$

$\Rightarrow \quad {}^{59}C_4 + {}^{59}C_5 \Rightarrow {}^{60}C_5$

Q(2) $+{}^{n}C_n + {}^{n+1}C_n + {}^{n+2}C_n + {}^{n+3}C_n + \cdots$

$\cdots + {}^{n+r}C_n$

$\Rightarrow$

$\boxed{{}^{n+1}C_{n+1} + {}^{n+1}C_n} + {}^{n+2}C_n + \cdots + {}^{n+r}C_n$

$\Rightarrow \quad {}^{n+2}C_{n+1} + {}^{n+2}C_n + {}^{n+3}C_n + \cdots + {}^{n+r}C_n$

$\Rightarrow \quad {}^{n+3}C_{n+1} + {}^{n+3}C_n + \cdots + {}^{n+r}C_n$

$\Rightarrow \quad {}^{n+r+1}C_{n+1}$

Q. ${}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + {}^{n+3}C_3 + \cdots + {}^{n+r}C_r$

$\Rightarrow$

Page No.
Date

☆ Type - III

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$$

put $x=1$ $\quad 2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$ — (i)

put $x=-1$ $\quad 0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + {}^nC_n(-1)^n$ — (ii)

(i) + (ii) $\quad 2^n = 2.({}^nC_0 + {}^nC_2 + {}^nC_4 + \dots)$

$$\therefore \boxed{2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + \dots}$$

(i) − (ii)

$$2^n = 2[{}^nC_1 + {}^nC_3 + {}^nC_5 + \dots]$$

$$\boxed{2^{n-1} = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots}$$

Q1// $\quad S = {}^nC_0 + 2.{}^nC_1 + 3.{}^nC_2 + 4.{}^nC_3 + \dots + (n+1){}^nC_n$

$\Rightarrow \quad S = (n+1){}^nC_n + n\,{}^nC_{(n-1)} + (n-1){}^nC_{n-2} + (n-2){}^nC_{n-3} + \dots + {}^nC_0$

Teacher's Signature.........

2021.06.11 17:0

$$2S = (n+2)\cdot {}^nC_0 + (n+2)\cdot {}^nC_1 + (n+2)\,{}^nC_2 + \cdots + (n+2)\,{}^nC_n$$

$$2S = (n+2)\left({}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n\right)$$

$$2S = (n+2)\cdot 2^n$$

$$\boxed{S = (n+2)\cdot 2^{n-1}}$$

Q. $S = {}^nC_0 + 4\cdot {}^nC_1 + 7\cdot {}^nC_2 + \cdots + (3n+1)\,{}^nC_n$

$\Rightarrow$ $S = (3n+1)\,{}^nC_n + (3n-2)\,{}^nC_{n-1} + \cdots\cdots {}^nC_0$

$$2S = (3n+2)\,{}^nC_n + (3n+2)\,{}^nC_{n-1} + (3n+2)\,{}^nC_{n-2}$$

$$2S = 3n+2\left({}^nC_n + {}^nC_{n-1} + {}^nC_{n-2} + \cdots\right.$$

$$2S = (3n+2)\left({}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n\right)$$

$$2S = (3n+2)\cdot 2^n$$

$$S = (3n+2)\cdot 2^{n-1}$$

Q3. $S = 0\cdot{}^nC_0 + {}^nC_1 + 2\cdot {}^nC_2 + 3\cdot {}^nC_3 + \cdots\cdots + n\cdot {}^nC_n$

$\Rightarrow$ $S = n\cdot {}^nC_n + (n-1)\,{}^nC_{n-1} + (n-2)\,{}^nC_{(n-2)} + \cdots\cdots 0\cdot {}^nC_0$

Teacher's Signature..........

Page No.
Date

$2S = (n+0)\,{}^nC_n + (n+0)\,{}^nC_{n-1} + \cdots$

$2S = (n+0)\left({}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n\right)$

$2S = n\cdot(2^n)$

$\boxed{S = n\left(2^{n-1}\right)}$

* $\boxed{\sum_{r=0}^{n} r\cdot{}^nC_r = n\cdot 2^{n-1}}$

Proof :→

$\sum_{r=0}^{n} r\cdot{}^nC_r$

$\Rightarrow \sum r\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1}$

$\Rightarrow n\sum_{r=1}^{n} {}^{n-1}C_{r-1}$

$\Rightarrow n\cdot\left[{}^{n-1}C_0 + {}^{n-1}C_1 + \cdots + {}^{n-1}C_{n-1}\right]$

$\Rightarrow n\cdot 2^{n-1}$

Q. $\sum_{r=0}^{n} (r+1)\cdot{}^nC_r$

$\Rightarrow \sum (r+1)\,\frac{n}{(r+1)}$

$\Rightarrow \sum_{r=0}^{n}\left(r\cdot{}^nC_r + {}^nC_r\right)$

Teacher's Signature..........

$$\Rightarrow \sum_{r=0}^{n} r \cdot {}^{n}C_r + \sum_{r=0}^{n} {}^{n}C_r$$

$$\Rightarrow (n \cdot 2^{n-1} + 2^n)$$

Q. $$\sum_{r=0}^{n} (2r^2 + 7r + 5)\, {}^{n}C_r$$

$$\Rightarrow \sum_{r=0}^{n} (r+5)(r+2)\, {}^{n}C_r$$

$$\sum_{r=0}^{n} \left(r \cdot {}^{n}C_r + \sum_{r=0}^{n} 5 \cdot {}^{n}C_r \right)\left(r \cdot {}^{n}C_r + 2\,{}^{n}C_r \right)$$

$$\Rightarrow n \cdot 2^{n-1} \quad 5\,(2^n) \cdot (2n^{-1}) + 2(2^n)$$

$$\Rightarrow (n \cdot 2^{n-1}) \quad 5(2)^n \cdot 2n\,(2^{n-1}) + 2(2)^n$$

$$\Rightarrow 2 \cdot \sum_{r=0}^{n} r^2 \cdot {}^{n}C_r + 7 \cdot \sum_{r=0}^{n} r \cdot {}^{n}C_r + 5 \sum_{r=0}^{n} {}^{n}C_r$$

$$\Rightarrow \sum_{r=0}^{n} r^2 \cdot {}^{n}C_r = \sum r^2 \cdot \frac{n}{\cancel{r}} \cdot {}^{n-1}C_{r-1}$$

$$= n\left(\sum (r-1+1)\, {}^{n-1}C_{r-1} \right)$$

$$\Rightarrow n\left[\sum (r-1)\, {}^{n-1}C_{r-1} + \sum {}^{n-1}C_{r-1} \right]$$

Page No.
Date

$$^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$$

$$\Rightarrow n\left[\sum (r-1) \frac{n-1}{(r-1)} \cdot {}^{n-2}C_{r-2} + \sum {}^{n-1}C_{r-1}\right]$$

$$\Rightarrow n\left[(n-1) \sum_{r=2}^{n} {}^{n-2}C_{r-2} + \sum_{r=1}^{n} {}^{n-1}C_{r-1}\right]$$

$$\Rightarrow n\left[(n-1)\{{}^{n-2}C_0 + {}^{n-2}C_1 + \dots + {}^{n-2}C_{n-2}\} + \{{}^{n-1}C_0 + \dots + {}^{n-1}C_{n-1}\}\right]$$

$$\Rightarrow n\left[(n-1)\, 2^{n-2} + 2^{n-1}\right]$$

$$\Rightarrow n \cdot 2^{n-2}\left[n-1+2\right] = n(n+1) \cdot 2^{n-2}$$

☆ Type - IV :- $\;{}^nC_{r_1} \cdot {}^nC_{r_2}$

$$r_1 + r_2 = \text{Constant}$$
$$|r_1 - r_2| = \;\;"$$

Q1 $\;{}^{30}C_0 \cdot {}^{40}C_{20} + {}^{30}C_1 \cdot {}^{40}C_{19} + {}^{30}C_2 \cdot {}^{40}C_{18} + \dots$
$\dots {}^{30}C_{20} \cdot {}^{40}C_{0}$.

$$\Rightarrow {}^{70}C_{20}$$

Q2 $\;{}^nC_0 \cdot {}^nC_n + {}^nC_1 \cdot {}^nC_{n-1} + {}^nC_2 \cdot {}^nC_{n-2} + \dots$
$\dots + {}^nC_n \cdot {}^nC_0$

Teacher's Signature..........

2021.06.11 17:00

$= {}^{2n}C_n$

(3) ${}^{40}C_0 \cdot {}^{40}C_{18} + {}^{40}C_1 \cdot {}^{40}C_{17} + {}^{40}C_2 \cdot {}^{40}C_{16} + \dots + {}^{40}C_{18} \cdot {}^{40}C_0$

$\Rightarrow {}^{80}C_{18}$

(4) ${}^{30}C_0 \cdot {}^{30}C_2 + {}^{30}C_1 \cdot {}^{30}C_{27} + \dots + {}^{30}C_{28} \cdot {}^{30}C_0$

$\Rightarrow {}^{30}C_0 \cdot {}^{30}C_{28} + {}^{30}C_1 \cdot {}^{30}C_{27} + \dots + {}^{30}C_{28} \cdot {}^{30}C_0$

$\Rightarrow {}^{60}C_{28}$

(5) ${}^{20}C_0 \cdot {}^{30}C_6 + {}^{20}C_1 \cdot {}^{30}C_5 + {}^{20}C_2 \cdot {}^{20}C_6 \dots$

$\dots + {}^{20}C_{20} \cdot {}^{30}C_{24}$

$\Rightarrow {}^{20}C_0 \cdot {}^{30}C_{26} + {}^{20}C_1 \cdot {}^{30}C_{25} + \dots$

$+ {}^{20}C_{20} \cdot {}^{30}C_6$

$\Rightarrow {}^{50}C_{26}$

(6) ${}^{n}C_0 \cdot {}^{n}C_0 + {}^{n}C_1 \cdot {}^{n}C_1 + {}^{n}C_2 \cdot {}^{n}C_2 + \dots + {}^{n}C_n \cdot {}^{n}C_n$

$\Rightarrow {}^{n}C_0 \cdot {}^{n}C_n + {}^{n}C_1 \cdot {}^{n}C_{n-1} + {}^{n}C_2 \cdot {}^{n}C_{n-2} + \dots$
$+ {}^{n}C_n \cdot {}^{n}C_0$

$\Rightarrow {}^{2n}C_n$

Teacher's Signature..........

2021.06.11 17:0

Page No.
Date

Q. Subjective approach :-

(for Q 1)

$(1+n)^{30} = {}^{30}C_0 + {}^{31}C_1 n + \cdots + {}^{30}C_{18} n^{18} + {}^{30}C_{19} n^{19}$

$\cdots + {}^{30}C_{30} n^{30}$

$(1+n)^{40} = {}^{40}C_0 + {}^{40}C_1 n + \cdots + {}^{40}C_{18} n^{18} + {}^{40}C_{19} n^{19}$

$\cdots + {}^{40}C_{40} n^{40}$

multiply

$(1+n)^{30} \cdot (1+n)^{40} = \cdots + ({}^{30}C_0 \cdot {}^{40}C_{20} + {}^{30}C_1 \cdot {}^{40}C_{19}$
$+ {}^{30}C_2 \cdot {}^{40}C_{18} + \cdots + {}^{30}C_{20} \cdot {}^{40}C_{20}) n^{20} + \cdots$

${}^{30}C_0 \cdot {}^{40}C_{20} + {}^{30}C_1 \cdot {}^{40}C_{19} + \cdots + {}^{30}C_{20} \cdot {}^{40}C_0 =$

Coeff of n^{20} in $(1+n)^{30}(1+n)^{40}$

$\Rightarrow$ $(1+n)^{70}$

$\Rightarrow {}^{70}C_{20}$

for (Que 5)

$(1+n)^{20} = {}^{20}C_0 + {}^{20}C_1 n + \cdots + {}^{20}C_{19} n + {}^{20}C_{20}$

$(1+n)^{30} \Rightarrow {}^{30}C_0 + {}^{30}C_1 n + \cdots + {}^{30}C_5 n^5 + {}^{30}C_6 n^6$
$+ \cdots + {}^{30}C_{24} n^{24} + \cdots + {}^{30}C_{30} n^{30}$

Teacher's Signature..........

2021.06.11 17:00

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multiply $(1+x)^{20}(1+x)^{30} = \cdots + \left({}^{20}C_0 \cdot {}^{30}C_{26} + {}^{20}C_1 \cdot {}^{30}C_{25} + \cdots + {}^{20}C_{20} \cdot {}^{30}C_6\right) x^{26} \cdots$

Compare coeff

$${}^{20}C_0 \cdot {}^{30}C_{26} + {}^{20}C_1 \cdot {}^{30}C_{25} + \cdots + {}^{20}C_{20} \cdot {}^{30}C_6$$

$\Rightarrow$ Coeff. of x^{26} in $(1+x)^{20} \cdot (1+x)^{30} = (1+x)^{50}$

$\Rightarrow {}^{50}C_{26}$

Type-V Differentiation and Integration approach:

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 \cdots + C_nx^n$$

Diff w.r.t x

$$n(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + \cdots + nC_nx^{n-1}$$

put $x = 1$

$$n(2)^{n-1} = C_1 + 2C_2 + 3C_3 \cdots + nC_n = \sum_{r=0}^{n} r\,{}^nC_r$$

put $x = -1$ $\quad 0 = C_1 - 2C_2 + 3C_3 - 4C_4 \cdots + nC_n(-1)^{n-1}$

Q. $C_0 + 2C_1 + 3C_2 + 4C_3 + \cdots + (n+1)C_n = ?$

$\Rightarrow$ (x) by (x)

$$x(1+x)^n = C_0x + C_1x^2 + C_2x^3 + \cdots + C_nx^{n+1}$$

2021.06.11 17:0

Page No.
Date

put $x = 1$

$n(2)^{n-1} + (2)^n \Rightarrow C_0 + 2C_1 + 2C_2 + 2C_3 + \dots + (n+1)C_n$

Q. $C_0 + 3C_1 + 5C_2 + 7C_3 + \dots + (2n+1)C_n = ?$

$\Rightarrow C_0 + 3C_1 + 5C_2 + 7C_3 + \dots + (2n+1)C_n =$

$\boxed{(2n+2)(2^{n-1})}$

(x) by (x)

$x(1+x^2)^n = C_0 x + C_1 x^3 + C_2 x^5 + C_3 x^7 \dots + C_n x^{2n+1}$

Diff. w.r.t to (x)

$x \cdot n(1+x^2) \cdot 1 \cdot (1+x^2)^n$

$\cdot (2n+2)2^{n-1} = C_0 + 3C_1 + 5C_2 + 7C_3 \dots (2n+1)C_n$

Q. $\sum r^2 \cdot {}^nC_r = 1^2 C_1 + 2^2 C_2 + 3^2 C_3 + \dots + n^2 C_n$

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$

Diff w.r.t (x)

$n(1+x)^{n-1} = C_1 + 2C_2 x + 3C_3 x^2 + \dots$

Teacher's Signature

2021.06.11 17:0

$- - + {}^nC_n x^{n-1}$

(x) by (n)

$$n x (1+x)^{n-1} = C_1 x + 2C_2 x^2 + \dots + nC_n x^n$$

diff. w.r.t (x)

$$1(x(1+x)^{n-1}) \; n(1+x)^{n-2} + nx(2)^{n-1}$$

$$x(1+x)^{n-1} + nx(2)^{n-2}$$

Put $x=1$

$$n(2^{n-1} + (n-1)(2)^{n-2}$$

Q. $(1+x)^n = C_0 + C_1 x + C_2 x^2 + - - - + C_n x^n$

$$\int_0^b (1+x)^n dx = \int_0^b (C_0 + C_1 x + \dots + C_n x^n) dx$$

$$\Rightarrow \left[C_0 x + \frac{C_1}{2} x^2 + \frac{C_2}{3} x^3 + - - + \frac{C_n}{n+1} x^{n+1}\right]_0^b$$

Q. $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} + - - - - \frac{C_n}{n+1}$

$$\int_0^1 (1+x)^n dx = \left[C_0 x + \frac{C_1 x^2}{2} + - - - + \frac{C_n}{n+1} x^{n+1}\right]_0^1$$

$$\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + - - - + \frac{C_n}{n+1} - 0$$

2021.06.11 17:00

Teacher's Signature..........

Page No.
Date

$$\Rightarrow \frac{2^{n+1}}{n+1} - \frac{1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots + \frac{C_n}{n+1}$$

Q. $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \cdots$

$$\Rightarrow C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \cdots$$

$$\int_0^1 (1-x)^n \, dx = \left[C_0 x - \frac{C_1 x^2}{2} + \cdots + \frac{(-1)^n C_n x^{n+1}}{n+1} \right]_0^1$$

$$\frac{(1-x)^{n+1}}{n+1} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \cdots$$

put $x=1$

$$-\left[\frac{(1-x)^{n+1}}{n+1} \right]_0^1$$

$$-\cancel{\frac{(2^{n+1})}{n+1}}$$

Q. $3C_0 + \frac{3^2}{2} C_1 + \frac{3^3}{3} C_2 + \frac{3^4}{4} C_3$

$$\Rightarrow \int_0^3 (1+x)^n \, dx = \int_0^3 \left(C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n \right) dx$$

$$\left[\frac{(1+x)^{n+1}}{n+1} \right]_0^3 \Rightarrow 3C_0 + \frac{3^2}{2} x + \cdots$$

Teacher's Signature............

$$\frac{(4)^{n+1}}{n+1} = 3C_0 + \frac{3^2}{2}C_1 n + \frac{3^3}{3}C_2 n^2 + \dots$$

$$\dots + \frac{3^{n+1}}{n+1}C_n \quad (1)$$

Q If $(1+x+x^2)^n = a_0 + a_1 x + a_2 x^2 + \dots$
$\dots + a_{2n} x^{2n}$

then

(i) $a_0 + a_1 + a_2 + a_3 + \dots + a_{2n} = 3^n$

$\Rightarrow$ put $x = 1$

$$(1+1+1)^n = a_0 + a_1 + a_2 + \dots + a_{2n}$$

$$3^n = a_0 + a_1 + a_2 + a_3 + \dots + a_{2n}$$

(ii) $a_0 - a_1 + a_2 - a_3 + \dots + a_{2n} = 1$

put $x = -1$

$$1 = a_0 - a_1 + a_2 - a_3 + \dots + a_{2n} \quad (ii)$$

(iii) $a_0 + a_2 + a_4 + \dots + a_{2n} = \frac{3^n+1}{2}$

$\Rightarrow$ (i) + (ii)

$$3^n + 1 = 2(a_0 + a_2 + a_4 + \dots + a_{2n})$$

$$\frac{3^n+1}{2} = a_0 + a_2 + a_4 + \dots + a_{2n}$$

Page No.
Date

(iv) $a_1 + a_3 + a_5 + \cdots + a_{2n-1} = \frac{3^n - 1}{2}$

$\Rightarrow$ (I) $-$ (II)

$$3^n - 1 = 2(a_1 + a_3 + a_5 + \cdots + a_{2n-1})$$

$$\frac{3^n - 1}{2} = a_1 + a_3 + a_5 + \cdots + a_{2n-1}$$

Note:- (i) To find sum of coefficient of even powers of x. Put $x = 1$ & $x = -1$ and add both expression.

(ii) To find sum of coefficient of odd powers of x. Put $x = 1$ & $x = -1$ and substract both expression.

(v) find coefficient of x^6 in the expension $(1 + x^2 + x^4)^n$

$\Rightarrow$ put $x = x^2$

$$(1 + x^2 + x^4)^n = a_0 + a_1 x^2 + \cdots + a_{2n} x^{4n}$$

coeff. of $x^6 = a_3$

(vi) find cofficient of x^{11} in the expension of $(1 + x^2 + x^4)^n$

Teacher's Signature..........

Page No.
Date

coeff of $x^{11} = 0$

put $x = x^2$

$$(1+x^2+x^4)^n = a_0 + a_1 x^2 + a_2 x^4 + \dots + a_{2n} x^{4n}$$

(vii) P.T

$$a_r = a_{2n-r} \quad \forall \quad 0 \le r \le n$$

$\Rightarrow$ Put $x = \frac{1}{x}$

(x) by x^{2n} $\left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^n = a_0 + \frac{a_1}{x} + \frac{a_2}{x_2} + \dots + \frac{a_{2n}}{x^2}$

$$(x^2+x+1)^n = a_0 x^{2n} + a_1 x^{2n-1} + a_2 x^{2n-2} + \dots + a_{2n}$$

$$a_0 = a_{2n}$$
$$a_1 = a_{2n-1}$$
$$a_2 = a_{2n-2}$$
$$\vdots$$
$$\boxed{a_r = a_{n-r}}$$

(viii) P.T

$$a_0^2 - a_1^2 + a_2^2 - a_3^2 + \dots + a_{2n}^2 = a_n$$

$\Rightarrow$ put $x = \frac{-1}{x}$

Teacher's Signature..........

Page No.
Date

(x) by 2^{2n} $\left(1-\frac{1}{n}+\frac{1}{n^2}\right)^n = a_0 - \frac{a_1}{n} + \frac{a_2}{n^2} - \frac{a_3}{n^3} + \cdots + \frac{a_{2n}}{n^{2n}}$

$(n^2 - n + 1)^n = a_0 n^{2n} - a_1 n^{2n-1} - \cdots + a_{2n}$

(i)(ii) $(1+n+n^2)^n \cdot (n^2-n+1)^n = \cdots \cdots (a_0^2 + a_1^2 + a_2^2 + \cdots + a_{2n}^2) + \cdots$

Compaire cofficient of n^{2n}

$a_0^2 - a_1^2 + a_2^2 - a_3^2 + \cdots + a_{2n}^2$ = coefy of n^{2n} in $(1+n+n^2)^n (n^2-n+1)^n$

$\longrightarrow ((1+n^2)+n)(n^2+1-n)^n$

$\longrightarrow ((1+n)^2 - n^2)^n$

$\longrightarrow (1+n^2+n^4)^n$

$\Longrightarrow a_n$

(ix) P.T $a_0 a_1 - a_1 a_2 + a_2 a_3 + \cdots - a_{2n-1} + a_{2n} = 0$

Teacher's Signature..........

2021.06.11 17:0

Scanned with CamScanner

Page No.
Date

1 x (ii)

$\Rightarrow (1+x+x^2)^n \cdot (x^2-x+1)^n = \dots\dots + (a_0a_1 - a_1a_2 + a_2a_3 + \dots - a_{2n-1}a_{2n}) x^{2n+1}$

$a_0a_1 - a_1a_2 + \dots - a_{2n-1}a_{2n} = $ coeff. of x^{2n+1} in $(1+x+x^2)^n \cdot (x^2-x+1)$

$= ((1+x^2)+x)(x^2+1-x)^n$

$= ((1+x)^2 - x^2)^n$

$= (1+x^2+x^4)^n$ coeff of x^{2n+1} in

$\Rightarrow 0$

(X) P.T

$a_0a_2 = a_1a_3 + a_2a_4 + \dots + a_{2n-2} \cdot a_{2n}$

$= a_{n-1}$

$\Rightarrow$ Put $x = \frac{1}{x}$

(1) x (ii) $(1+x+x^2)^n \cdot (x^2-x+1)^n = \dots + (a_0a_2 - a_1a_3 + a_2a_4 - a_3a_5 \dots) x^{2n-2}$

Compare coefficient of x^{2n+2} in

$(1+x+x^2)^n (x^2-x+1)^n$

Teacher's Signature..........

2021.06.11 17:0

Page No.
Date

$$\Rightarrow \left((1+x^2)+x\right)(1+x^2-x)^n$$

$$\Rightarrow \left((1+x^2)^2-x^2\right)^n$$

$$\Rightarrow \left(1+x^2+x^4\right)^n$$

Coff. of $x^{2n-2} \Rightarrow a_{n-1}$

Q. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, find $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$$\Rightarrow S = \frac{0}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \dots + \frac{n}{{}^nC_n}$$

$$S = \frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \frac{n-2}{{}^nC_{n-2}} + \dots + \frac{0}{{}^nC_0}$$

add $$2S = \frac{n}{{}^nC_0} + \frac{n}{{}^nC_1} + \frac{n}{{}^nC_2} + \dots + \frac{n}{{}^nC_n}$$

$$2S = n\left(\frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \dots + \frac{1}{{}^nC_n}\right)$$

$$\therefore 2S = n \cdot a_n$$

$$\boxed{S = \frac{n \cdot a_n}{2}}$$

Teacher's Signature.........

Scanned with CamScanner

Page No.
Date

Q. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$

and $a_k = 1 \quad \forall \; k \geq n$

P.T $\quad b_n = {}^{2n+1}C_{n+1}$

$\Rightarrow$ put $(x-3) = t$

$$\sum_{r=0}^{2n} a_r (1+t)^r = \sum_{r=0}^{2n} b_r t^r$$

$$\Rightarrow a_0 (1+t)^0 + a_1 (1+t)^1 + \dots + a_{n-1} (1+t)^{n-1} + a_n (1+t)^n + a_{n+1} (1+t)^{n+1} + \dots + a_{2n} (1+t)^{2n}$$

$$\Rightarrow b_0 + b_1 t + b_2 t^2 + \dots + b_n t^n + b_{n+1} t^{n+1} + \dots + b_{2n} t^{2n}$$

$b_n =$ coeff. of t^n in $a_n (1+t)^n + a_{n+1} (1+t)^{n+1} + \dots + a_{2n} (1+t)^{2n}$

$$a_n = a_{n+1} = a_{n+2} \dots = a_{2n} = 1$$

Coeff of t^n in

$$\left[(1+t)^n + (1+t)^{n+1} + \dots + (1+t)^{2n} \right]$$

Teacher's Signature.............

2021.06.11 17:00

Scanned with CamScanner

Page No.
Date

$$b_n = {}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n - - - + {}^{2n}C_n$$

$$\Rightarrow {}^{n+1}C_{n+1} + {}^{n+1}C_n + {}^{n+2}C_n + {}^{n+3}C_n$$

$$+ - - - - - - + {}^{2n}C_n$$

$$\Rightarrow {}^{n+2}C_{n+1} + {}^{n+2}C_n + {}^{n+3}C_n + - - - + {}^{2n}C_n$$

$$\Rightarrow {}^{n+3}C_{n+1} + {}^{n+3}C_n + - - - - + {}^{2n}C_n$$

$$\Rightarrow {}^{2n+1}C_{n+1}$$

Q. $1({}^nC_0)^2 + 3.({}^nC_1)^2 + 5.({}^nC_2)^2 + - - -$
$+ (2n+1)({}^nC_n)^2 = ?$

$$\Rightarrow S = 1({}^nC_0)^2 + 3.({}^nC_1)^2 + 5.({}^nC_2)^2 + - - - + (2n+1)({}^nC_n)^2$$

$$\Rightarrow S = (2n+1)({}^nC_n)^2 + (2n-1)({}^nC_{n-1})^2 + (2n-3)({}^nC_{n-2})^2$$
$$+ - - - - - - ({}^nC_0)^2$$

$$2S = (2n+2)({}^nC_0)^2 + (2n+2)({}^nC_1)^2 + - - -$$
$$- - - + ({}^nC_n)^2$$

$$2S = (2n+2)\left(({}^nC_0)^2 + ({}^nC_1)^2 + \dots + ({}^nC_n)^2\right)$$

Teacher's Signature..........

Page No.
Date / /

$$2S = (2n+2)\left(2\,{}^{2n}C_n\right)$$

$$2S = (2n+2)\left({}^{2n}C_n\right)$$

$$S = (n+1)\,{}^{2n}C_n$$

Q. P.T

$$\left(\frac{{}^nC_0+{}^nC_1}{{}^nC_0}\right)\cdot\left(\frac{{}^nC_1+{}^nC_2}{{}^nC_1}\right)\cdots\cdots\left(\frac{{}^nC_{n-1}+{}^nC_n}{{}^nC_{n-1}}\right) = \frac{(n+1)^n}{\lfloor n}$$

$$\Rightarrow \quad \frac{{}^nC_0+{}^nC_1}{{}^nC_0} = \frac{1+n}{1}$$

$$\frac{{}^nC_1+{}^nC_2}{{}^nC_1} = \frac{n+\frac{n(n-1)}{1.2}}{n} = \frac{n+1}{2}$$

$$\frac{{}^nC_2+{}^nC_3}{{}^nC_2} = \frac{\frac{n(n-1)}{2}+\frac{n(n-1)(n-2)}{\lfloor 3}}{\frac{n(n-1)}{2}}$$

$$\Rightarrow \frac{1+n}{3}$$

$$\frac{{}^nC_{n-1}+{}^nC_n}{{}^nC_{n-1}} = \frac{1+n}{n}$$

$$L.H.S = \left(\frac{1+n}{1}\right)\left(\frac{1+n}{2}\right)\left(\frac{1+n}{3}\right)\cdots\left(\frac{1+n}{n}\right)$$

Teacher's Signature............

2021.06.11 17:0

$$= \frac{(1+n)^n}{\lfloor n}$$

OR

$$\prod_{r=1}^{n} \left(\frac{{}^nC_{r-1} + {}^nC_r}{{}^nC_{r-1}} \right) = \frac{(n+1)^n}{n!}$$

$$L.H.S = \prod_{r=1}^{n} \left(\frac{{}^{n+1}C_r}{{}^nC_{r-1}} \right)$$

$$= \prod_{r=1}^{n} \left[\frac{\overset{(n+1)}{\cancel{\lfloor n+1}}}{\cancel{\lfloor n-r+1}\, \cancel{\lfloor r}\, r} \times \frac{\cancel{\lfloor n-r+1}\, \cancel{\lfloor r-1}}{\cancel{\lfloor n}} \right]$$

$$\Rightarrow \prod_{r=1}^{n} \left(\frac{1+n}{r} \right)$$

$$\Rightarrow \left(\frac{1+n}{1} \right) \left(\frac{1+n}{2} \right) \left(\frac{1+n}{3} \right) \cdots \left(\frac{1+n}{n} \right)$$

$$\Rightarrow \frac{(1+n)^n}{\lfloor n}$$

* Binomial Theorem for any index

2021.06.11 17:00

$n \in Q$, $|x| < 1$

★ $(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \frac{n(n-1)(n-2)}{\lfloor 3}x^3$

$+ - - - - - \infty$

$(1-x)^n = 1 - nx + \frac{n(n-1)}{\lfloor 2}x^2 - \frac{n(n-1)(n-2)}{\lfloor 3}x^3$

$+ - - - - - \infty$

$(1+x)^{-n} = 1 - nx + \frac{n(n+1)}{\lfloor 2}x^2 - \frac{n(n+1)(n+2)x^3}{\lfloor 3}$

$+ - - - - \infty$

★ $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{\lfloor 2}x^2 + \frac{n(n+1)(n+2)}{\lfloor 3}x^3$

$- - - \infty$

Q. for what values of x expension of $(8+3x)^{1/2}$ is valid.

$\Rightarrow (8+3x)^{1/2}$

$\Rightarrow 8\left(\cancel{8} + \frac{3x}{8}\right)^{-1/2}$

$\Rightarrow 8^{-1/2}\left(1+\frac{3x}{8}\right)^{1/2}$ $\quad -1 < \frac{3x}{8} < 1$

Page No.
Date

$$x \in \left(-\frac{8}{3}, \frac{8}{3}\right)$$

(ii) find coefficient of x^7.

$\Rightarrow$ Coeff of x^7 in $(1+x)^n$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{\lfloor 7}$$

Coeff of x^7 in $(8+3x)^{-1/2}$.

$$8^{-1/2}\left(\frac{3x}{8}\right)^{7} \frac{\left(\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\cdots\left(-\frac{1}{2}-6\right)\right)}{\lfloor 7}$$

put $x = 1$

Q. $(1-x)^{-1} =$

$1 < 1-x < 1$

$-x+1 > 1$

$x-1 < 1$

$x < 2$

$x \in (-\infty, 2)$

$-x+1 < 1$

$x-1 > 1$

$x > 2$

$x \in (2, \infty)$

Coeff of x^7 in $(1-x)^{-1}$

$$-1\frac{\left((-1)(-1-1)(-1-2)\cdots(-1-6)\right)}{\lfloor 7}$$

Teacher's Signature..........

2021.06.11 17:00

Page No.
Date

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots \infty$

Q. $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots \infty$

Note :-> for $n \in N$

$(1-x)^{-n} = 1 + {}^{n}C_1 x + {}^{n+1}C_2 x^2 + {}^{n+2}C_3 x^3 + \cdots \infty$

$$\boxed{\text{Coefficient of } x^r \text{ in } (1-x)^{-n} = {}^{n+r-1}C_r}$$

Q. find Coefficient of x^5 in $(1-x)^{-3}$

x^5 $(1-x)^{-3}$ $= {}^{3+5-1}C_5$

$\Rightarrow {}^{7}C_5$

$\Rightarrow {}^{7}C_5$

Q. find coefficient of x^{10} in $\left(2 + 3x - \frac{5}{x}\right)(1-x)^{-3}$

0	10	$2 \times {}^{3+10-1}C_{10} = 2 \times {}^{12}C_{10}$
1	9	$3 \times {}^{3+9-1}C_9 = 3 \times {}^{11}C_9$
-1	11	$-5 \times {}^{3+11-1}C_{11} = 3 \times {}^{12}C_{11}$

Teacher's Signature.........

2021.06.11 17:00

Page No.
Date

Q. Find coefficient of x^{11} in $(1+x+x^2+\dots+x^{15})^2 (1+x+x^2+\dots x^{10})^3$

$(1+x+x^2+\dots+x^{15})^2 (1+x+x^2+\dots+x^{10})^3$

$\Rightarrow \left(\frac{(1-x^{16})}{1-x}\right)^2 \left(\frac{1(1-x^{11})}{1-x}\right)^3$

$\Rightarrow \frac{(1-x^{16})^2 (1-x^{11})^3}{(1-x)^5}$

$\Rightarrow (1-x^{16})^2 (1-x^{11})^3 (1-x)^{-5}$

$\Rightarrow (1-2x^{16}+x^{32})(1-3x^{11}+3x^{22}-x^{33})(1-x)^{-5}$

$\Rightarrow (1-3x^{11}+3x^{22}-x^{33})(1-x)^{-5}$

0	11	$1\times{}^{5+11-1}C_{11}$
11	0	-3×1

$\Rightarrow {}^{15}C_{11} - 3$

* Coefficient Method :-

Q. $A+B=5$
$0\le A\le 6$, $0\le B\le 4$, find no. of integral Soln.

A	B
1	4
2	3
3	2
4	1
5	0

Ans = 5

Teacher's Signature..........

M-II No. of Sol^n

Coeff of x^5

$(x^0+x^1+x^2+x^3+x^4+x^5+x^6)(x^0+x^1+x^2+x^3+x^4)$

Q. $P+C+M=150$

$0 \le P \le 50$, $0 \le C \le 100$, $0 \le M \le 150$, find no. of integral sol^n.

$\Rightarrow$ $1+2+147$

$40+30+80$

Coeff of x^{150} in expansion of

$(x^0+x^1+x^2+\dots+x^{50})(x^0+x^1+x^2+\dots+x^{100})(x^0+x^1+\dots+x^{150})$

Q. $P+C+M=100$

$10 \le P \le 70$, $10 \le C \le 70$, $10 \le M \le 70$

find no. of integral sol^n.

$\Rightarrow$ $(x^{10}+x^{11}+\dots+x^{70})(x^{10}+x^{11}+\dots x^{70})$

$(x^{10}+x^{11}+x^{12}+\dots x^{70})$

Coeff of x^{100}

$\Rightarrow$ $(x^{10}+x^{11}+\dots x^{70})^3$

2021.06.11 17:00

Page No.
Date

Q. $P(40)$, $C(40)$, $M(40)$

Total = 70

atleast 20 marks in chemistry.

$\Rightarrow$ Coff of x^{70} in $(x^0+x^1+\cdots x^{40})^2(x^{20}+x^{21}+\cdots+x^{40})$

Q. $P(50)$, $C(50)$, $M(50)$, Total = 100

even marks in maths.

$\Rightarrow$ Coeff x^{100} $(x^0+x^1+x^2+\cdots x^{50})^2(x^0+x^2+x^4+x^6\cdots x^{50})$

Teacher's Signature............

2021.06.11 17:

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