Binomial Theorem

* **Binomial expression** :- Expression containing two dissimilar term's

* Trinomial expression :-> Expression containing 3 dissimilar term's

* Monomial :-> Expression containing 1 term.

* **Pascal** :-

$$(n+y) = n+y$$

$$(n+y)^2 = (n+y)(n+y) = n^2+2ny+y^2$$

$$(n+y)^3 = (n+y)(n+y)(n+y) = n^3+3n^2y+3ny^2+y^3$$

1 — 1

1 — 2 — 1

1 — 3 — 3 — 1

1 — 4 — 6 — 4 — 1

1 — 5 — 10 — 10 — 5 — 1

1 — 6 — 15 — 20 — 15 — 6 — 1

2021.06.11 17:00

Page No.
Date

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10y^3x^2 + 5xy^4 + y^5$

★ Newton's Binomial Theorem $(n \in N)$:-

$(x+y)^n = {}^nC_0 x^n + {}^nC_1 \cdot x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \cdots$
$\cdots + {}^nC_n y^n.$

$(x+y)^5 = {}^5C_0 x^5 + {}^5C_1 \cdot x^4 y + {}^5C_2 \cdot x^3 y^2 + {}^5C_3 x^2 y^3$
${}^5C_4 \cdot xy^4 + {}^5C_5 \cdot y^5$

$\Rightarrow x^5 + 5x^4y + 10x^3y^3 + 10x^2y^3 + 5xy^4$
$+ y^5$

Q (i) $(x-2y)^5$

$\Rightarrow {}^5C_0 x^5 + {}^5C_1 x^4(-2y) + {}^5C_2 x^3(-2y)^2$
$+ {}^5C_3 x^2(-2y)^3 + {}^5C_4 x(-2y)^4 + {}^5C_5(-2y)^5$

$\Rightarrow x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3$
$+ 80xy^4 - 32y^5$

(ii) $(1+x)^n \Rightarrow {}^nC_0 1 + {}^nC_1 1^{n-1} x + {}^nC_2 1^{n-2} \cdot x^2$
$+ {}^nC_3 1^{n-3} x^3 \cdots\cdots {}^nC_n 1^0 x^n$

2021.06.11 17:00

Page No.
Date

$\Rightarrow {}^nC_0 + {}^nC_1 x + {}^nC_2 \cdot x^2 + \cdots {}^nC_n \cdot x^n$

(iii) $(1-x)^n = {}^nC_0 + {}^nC_1 1^{n-1}(-x) + {}^nC_2 1^{n-2}(-x)^2$

$+ {}^nC_3 (-x)^n$

$\Rightarrow {}^nC_0 - {}^nC_1 + {}^nC_2 x^2 - {}^nC_3 x^3 \cdots + {}^nC_n(-1)^n x^n$

(iv) $(\sqrt{2}+1)^6 - (\sqrt{2}-1)^6$

$\Rightarrow {}^6C_0(\sqrt{2})^5 + {}^6C_1(\sqrt{2})^4 1 + {}^6C_2(\sqrt{2})^3(1)^2 + {}^6C_3(\sqrt{2})^2 + {}^6C_4$
$(\sqrt{2})^1 + {}^6C_5\sqrt{2} + {}^6C_5(\sqrt{2})^6 - [{}^6C_0(\sqrt{2})^6 - {}^6C_1(\sqrt{2})^5$
$+ {}^6C_2(\sqrt{2})^3 - {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^2 - {}^6C_5(\sqrt{2})^1 + {}^6C_5\sqrt{2}^0]$

$\Rightarrow 2[{}^6C_1(\sqrt{2})^5 + {}^6C_3(\sqrt{2})^3 + {}^6C_5\sqrt{2}]$

$\Rightarrow 2[6 \times 4\sqrt{2} + 20 \times 2\sqrt{2} + 6\sqrt{2}]$

$2[24\sqrt{2} + 40\sqrt{2} + 6\sqrt{2}] = 2 \times 70\sqrt{2}$

$\Rightarrow 140\sqrt{2}$

(v) $(2+\sqrt{3})^5 + (2-\sqrt{3})^5$

${}^5C_0(2)^5 + \cancel{{}^5C_1(2)^4(\sqrt{3})} + {}^5C_2(2)^3(\sqrt{3}) + \cancel{{}^5C_3(2)^2(\sqrt{3})}$
${}^5C_4(2)^1(\sqrt{3})^4 + \cancel{{}^5C_5(\sqrt{3})^5} + {}^5C_0(2)^5 + \cancel{{}^5C_1(2)^4(-\sqrt{3})}$
$+ {}^5C_2(2)^3(-\sqrt{3})^2 + \cancel{{}^5C_3(2)^2(-\sqrt{3})^3} + {}^5C_4(2)^1(-\sqrt{3})^4$
$+ \cancel{{}^5C_5(-\sqrt{3})^5}$

$\Rightarrow 32 + \cancel{160} + 240 + 90 + 32 + 240 + 90$

2021.06.11 17:00

Page No.
Date

$\Rightarrow 724$

☆ Highlights of B.T :-

(1) No. of term's in the expension of ~~(x+y)~~ $(x+y)^n$ is $(n+1)$

$$(x+y)^n = {}^nC_0 \cdot x^n + {}^nC_1 \cdot x^{(n-1)} y \ldots {}^nC_n y^n$$

Q. find no. of dissimilar term's in the expension of

$$(x_1 + x_2 + x_3 + x_4)^5$$

$$\Rightarrow x_1^{p_1} \cdot x_2^{p_2} \cdot x_3^{p_3} \cdot x_4^{p_4}$$

where

$$p_1 + p_2 + p_3 + p_4 = 5$$

(non-negative interalsoln)

$\Rightarrow$ (Beggar's method)

$$\Rightarrow {}^8C_3$$

(2) Sum of indices (power) of x & y in each term is n.

Teacher's Signature...........

2021.06.11 17:00

Page No.
Date

(3) $^nC_0, ^nC_1, ^nC_2 - - - ^nC_n$ are called binomial coefficients.

* (a) Difference b/w cofficent and Binomial coeff:

$(x-2y)^5 = 5C_0 x^5 + 5C_1 \cdot x^4(-2y) + 5C_2 \cdot x^3(-2y)^2 - -$

Binomial cofficent $= 5C_0 + 5C_1 + 5C_2 - - -$

Coeff: $\rightarrow$ $5C_0 \; -2\,5C_1 \; 4\,5C_2 - - - -$

(b) Sum of Binomial cofficents

* $\boxed{nC_0 + nC_1 + nC_2 + nC_3 - - - nC_n = 2^n}$

Proof:-

$(1+x)^n = nC_0 + nC_1 x + nC_2 x^2 + - - + nC_n x^n$

Put $x=1$

$2^n = nC_0 + nC_1 + nC_n - - -$

(c) To find sum of cofficent put variable=1

		Sum of binomial coeff	Sum of coeff.
Q(1)	$(x-2y)^5$	$(2)^5$	-1
(2)	$(2x-y)^4$	2^4	1
(3)	$(x+y)^6$	2^6	2^6

Teacher's Signature............

2021.06.11 17:00

Page No.
Date

(4) Binomial coefficients of the term equidistant from begining & last are equal.

(5) Coefficent of $(x)^r$ in the expension of $(1+x)^n$ is ^nC_r.

(6) $(x+y)^n = {}^nC_0 x^n + {}^nC_1 \cdot x^{n-1} \cdot y + {}^nC_2 x^{n-2} y^2 + \dots + {}^nC_n y^n$

$$\boxed{(x+y)^n = \sum_{r=0}^{n} {}^nC_r (x)^{n-r} y^r}$$

Q (1) $\sum_{r=0}^{7} {}^7C_r \, 2^{7-r} \cdot 3^r = (2+3)^7$
$= 5^7$

(2) $\sum_{r=0}^{n} (-1)^r \, {}^nC_r \, (10)^{n-r} \cdot (9)^r$

$\Rightarrow (10-9)^n$
$= (1)^n$
$= 1$

(3) $\sum_{r=0}^{7} {}^7C_r \, 3^r$ $(1+3)^7$
$\Rightarrow (4)^7$

Teacher's Signature

2021.06.11 17:00

Page No.
Date

(4) $\sum_{r=0}^{7} {}^7C_r \, 3^{7-r} = (3+1)^7$

$\Rightarrow (4)^7$ or $(2)^7$

(5)

(5) $\sum_{r=0}^{n} {}^nC_r = 2^n$

(6) $\sum_{r=0}^{n} {}^nC_r \cdot 3^{2r} \cdot 3^r \Rightarrow (4+\cancel{3})^n$

$\Rightarrow (7)^n$ or $(10)^n$

(7) $\sum_{r=0}^{n} {}^nC_r \cdot 2^r = (3)^n$

(8) $\sum_{r=1}^{n} {}^nC_r \cdot 2^r = (3)^n - 1$

$\sum_{r=0}^{n} {}^nC_r \cdot 2^r - ({}^nC_0 \, 2^0)$

(9) $\sum_{r=2}^{n} {}^nC_r \cdot 2^r = (3)^{n-2}$

$3^n - (1+2n)$

$= 3^n - (1+2n)$

2021.06.11 17:00

Teacher's Sign

Scanned with CamScanner

★ General Term :-

$(x+y)^n = {}^nC_0\, x^n + {}^nC_1 \cdot x^{n-1} + \dots + {}^nC_n y^n$

$$\boxed{T_{(r+1)} = {}^nC_r \cdot x^{n-r} \cdot y^r}$$

$(r+1)^{th}$ term

Q- In the expension of $(x-2y)^{10}$ find T_4

$T_{(3+1)} = {}^{10}C_3 \cdot x^{10-3} \cdot y^3$

$T_{(3+1)} \Rightarrow {}^{10}C_3 \cdot x^7 (y)^3$

Q. Find coefficent of T_4

$T_{(3+1)} = {}^{10}C_3 (-2)^3$

Q. In the expension of $\left(x^2 + \frac{3}{x}\right)^6$ find the term involving x^3.

$T_{r+1} = {}^6C_r \cdot (x^2)^{6-r} \cdot \left(\frac{3}{x}\right)^r$

$\Rightarrow {}^6C_r \cdot 3^r \cdot x^{12-2r} \cdot \frac{1}{x^6}$

2021.06.11 17:00

Page No.
Date

$T_{r+1} = {}^6C_r \cdot 3^r \cdot x^{12-3r}$

(i) for x^3

$12 - 3r = 3$

$3r = 9$

$\boxed{r = 3}$

$\therefore$ term involving x^3 :-

$\Rightarrow T_{3+1} = T_4 = {}^6C_3 \cdot 3^3 \cdot x^3$

$\therefore$ coefficient of x^3 = ${}^6C_3 \cdot 3^3$

$\Rightarrow$ ~~540~~

$\frac{6 \times 5 \times 4 \times \cancel{\lfloor 3}}{\lfloor 3 \cancel{\lfloor 3}} = 20 \times 27$

$\Rightarrow 540$

(iii) find the term independent of x.

$T_{r+1} = {}^6C_r \cdot 3^r \cdot x^{12-3r}$

$\therefore$ for term independ of x

$12 - 3r = 0$

$r = 4$

~~$T_{4+1} = T_5$~~

Teacher's 2021.06.11 17:00

Page No.
Date

(iv) find cofficent of n^2

$$T_{r+1} = 6C_r \cdot 3^r \cdot n^{12-3r}$$

$$12-3r = 2$$

$$r = \frac{10}{3}$$

$\boxed{\text{not possible}}$

Their is no such term hence cofficent of n^2 is zero.

Q. find 15th term from the last in the expension of $(n+2y)^{51}$

$\Rightarrow$

$(p+1)^{th}$ term from last $= (n-p+1)$ term from beginning

M-I

15^{th} from last $= T_{(51-14+1)}$

$\Rightarrow T_{38}$

M-II

$(2y+n)^{51}$ $\Rightarrow$ 15^{th} term from the last in the expension of $(n+2y)^{51}$ will be 15^{th} term from the begining.

Teacher's Signature............

2021.06.11 17:00

Page No.
Date

in the expension of $(2y+x)^{51}$.

* **Middle term:-**

Binomial coefficent of the middle term is the greatest binomial cofficient.

Q $(1+x)^{30}$

$^{30}C_r \Rightarrow r = 15$

Middle term $= T_{r+1} = T_{15+1} = T_{16}$

Q $(1+x)^{52}$

$^{52}C_r = \quad r = 26$

middle term $= T_{26+1} = T_{27}$

Q. $(1+x)^{21}$

Middle term $\begin{cases} T_{10+1} = T_{11} \\ T_{11+1} = T_{12} \end{cases}$

Q. $(1+x)^{37}$

Middle term $\begin{cases} T_{18+1} = T_{19} \\ T_{19+1} = T_{20} \end{cases}$

Teacher's Signature..........

2021.06.11 17:00

Scanned with CamScanner

Page No.
Date / /

Q. find no. of rational term's in the expansion of $\left(4^{1/3}+\frac{1}{5^{1/4}}\right)^{60}$

$$\left(4^{1/3}+\frac{1}{5^{1/4}}\right)^{60}$$

$$\Rightarrow T_{r+1} = {}^{60}C_r \left(4^{1/3}\right)^{60-r} \cdot \left(\frac{1}{5^{1/4}}\right)^{r}$$

$$\Rightarrow {}^{60}C_r \, (4)^{\frac{60-r}{3}} \cdot (5)^{\frac{-r}{4}}$$

$\frac{60-r}{3} = I,$ | $\frac{r}{4} = I,$

$r_1 = 0, 3, 6, 9, \ldots 60$ | $r = 0, 4, 8, 12, \ldots 60$

Common value of r (common A.P)

LCM (3, 4) = 12

$r = 0, 12, 24, 36, 48, 60$

No. of rational terms ⇒ 6
No. of irrational " ⇒ 55

Q. $\left(3^{1/2}+5^{1/4}\right)^{99}$

Teacher's Signature..........

2021.06.11 17:00

$$T_{r+1} = {}^{99}C_r \left(3^{1/2}\right)^{99-r} \left(5^{1/4}\right)^{r}$$

$$\Rightarrow {}^{99}C_r \, (3)^{\frac{99-r}{2}} \, (5)^{\frac{r}{4}}$$

$\frac{99-r}{2} = I$, | $\frac{r}{4} = I$,

$r = 0, 4, 8, 9\ldots 12, 1\ldots$

$r = 0, 2, 4, 6, 8, \text{-----} 99$

★ Numerically Greatest Term (NGT)

★ $(x+y)^n$

$$|T_{r+1}| > |T_r|$$

$$\left|\frac{T_{r+1}}{T_r}\right| > 1$$

$$\Rightarrow \left| \frac{{}^nC_r \cdot x^{n-r} \cdot y^r}{{}^nC_{r-1} \cdot x^{n-r+1} \cdot y^{r-1}} \right| > 1$$

$$\Rightarrow \left| \frac{n-r+1}{r} \cdot \frac{y}{x} \right| > 1$$

$(x+1)^2$	x^2	$2x$	1
$x=1$	1	2	1
$x=2$	4	4	1
$x=3$	9	6	1
$x=1/2$	$\frac{1}{4}$	1	1
$x=1/3$	$\frac{1}{9}$	$\frac{2}{3}$	1

Page No.
Date

$\Rightarrow \quad \frac{n-r+1}{r} \cdot \left|\frac{y}{x}\right| > 1$

$\frac{n-r+1}{r} > \left|\frac{x}{y}\right|$

$n+1 > r\left|\frac{x}{y}\right| + r$

$n+1 > r\left(1+\left|\frac{x}{y}\right|\right)$

$\therefore \quad r < \frac{n+1}{1+\left|\frac{x}{y}\right|} \Rightarrow p$

$\lvert T_{r+1}\rvert > \lvert T_r\rvert$	$\lvert T_{r+1}\rvert = \lvert T_r\rvert$	$\lvert T_{r+1}\rvert < \lvert T_r\rvert$
$r < p$	$r = p$	$r > p$

☆ Case-I :- If $p \neq$ integer

Ex $\quad p = 5.2 \qquad$ if $r > p$

$|T_1| < |T_2| < |T_3| < |T_4| < |T_5| < |T_6| > |T_7| > |T_8|$

$\Rightarrow T_6$ is NGT

Teacher's Signature............

Page No.
Date

$\Rightarrow T_{5+1}$ is NGT

$\Rightarrow \boxed{T_{[p]+1}}$ is NGT

* <u>Case II</u> :→ If $p =$ integer
$p = 4$

$|T_1| < |T_2| < |T_3| < |T_4| = |T_5| > |T_6| > |T_7|$

T_4 & T_5 are NGT

$\Rightarrow$ $\boxed{T_p \text{ and } T_{p+1} \text{ are NGT}}$

Q. In expension of $(3-2x)^9$, find NGT if $x = 1/2$

(1) $x = 1/2$

$$p = \frac{n+1}{1+\left|\frac{x}{y}\right|}$$

$$p = \frac{9+1}{1+\left|\frac{3}{-2x}\right|}$$

$$\Rightarrow \frac{10}{1+\left|\frac{3}{-2\times\frac{1}{2}}\right|} = \frac{10}{4} = 2.5$$

Teacher's Signature

2021.06.11 17:00

Page No.
Date

$[p] = 2$

$\therefore$ T_{2+1} is NGrT = T_3 is NGrT

(ii) $n = 1$

$$p = \frac{9+1}{1+\left|\frac{3}{2}\right|}$$

$$p = \frac{20}{5}$$

~~p = 10~~ $P = 4$

$T_{r+1} = T_{5}$ is NGrT & T_5 is NGrT

(iii) $$p = \frac{9+1}{1+\left|\frac{3}{\cancel{3} \times 2}\right|}$$

$$p = \frac{10}{1+\frac{3}{3}}$$

$$\frac{\cancel{30}}{\cancel{6}} \quad P = 5$$

$T_{5+1} = T_6$ is NGrT

Teacher's 2021.06.11 17:00

Scanned with CamScanner

Q If in expension of $\left(2+\frac{3x}{8}\right)^{10}$, T_4 is NGT. find range of x.

$$T_4 = \quad \boxed{p=3}$$

$$p = \frac{10+1}{1+\left|\frac{2}{\frac{3x}{8}}\right|}$$

$$p = \frac{11}{1+\frac{16}{3|x|}}$$

$$p = \frac{33|x|}{16+3|x|}$$

$\therefore$ T_4 is NGT

$$3 < p < 4$$

$$3 < \frac{33|x|}{16+3|x|} < 4$$

$$\frac{33|x|}{16+3|x|} > 3 \quad \text{and} \quad \frac{33|x|}{16+3|x|} < 4$$

$48 + 9|x| < 33|x|$ $\qquad$ $21|x| < 64$

$48 < 24|x|$

$\therefore$ $21|x| < 64$

$\therefore$ $|x| > 2$ $\qquad$ and $\qquad$ $|x| < \frac{64}{21}$

$x < -2$ and $x > 2$ $\qquad$ $-\frac{64}{21} < x < \frac{64}{21}$

Q. In expansion of $\left(\frac{x}{5} + \frac{2}{5}\right)^n$, T_9 has NGC find (n).

$\Rightarrow$ T_9 is NGT for $x = 1$

$p = 11 < p < 12.5$

Q. (1) $(1 - 2x + 3x^2)(1+x)^{10}$, find coff of x^5

0	5	$1 \times {}^{10}C_5$
1	4	$(-2) \times {}^{10}C_4$
2	3	$3 \times {}^{10}C_3$

sum = 192

Teacher's Signature..........

2021.06.11 17:00

40
×11
40
40

Page No.
Date

Q(2) $\left(2x + \frac{3}{x} + 5x^2\right)(1+x)^{12}$ find coeff of x^7

1	3	$2 \times {}^{12}C_3$
-1	5	$3 \times {}^{12}C_5$
2	2	$5 \times {}^{12}C_2$

$\Rightarrow 2 \times \frac{\cancel{12} \times 11 \times 10 \times \cancel{10}}{\cancel{10}\ \cancel{3 \times 2}}$

Sum $\Rightarrow$ 440 + 792 + 330 $\Rightarrow$ 1562

Q.3 $(1+x^2)^6 (1+x)^7$, find coeff. of x^5

~~1~~	0	5	$1 \times {}^7C_5$
	2	3	${}^6C_1 \times {}^7C_3$
	4	1	${}^6C_2 \cdot {}^7C_1$

Sum = 346

Q. $(2 + 3x + 4x^2)\left(2x^2 - \frac{7}{x}\right)^{10}$, find coeff. of x

$\Rightarrow$

0	1	$2 \times 0 = 0$	calculation
1	0	$3 \times 0 = 0$	
2	-1	$4 \times {}^{10}C_7 (2)^3 (-7)^7$	$T_{r+1} = {}^{10}C_r (2x^2)^{10-r} \left(\frac{-7}{x}\right)^r$

Sum =

Teacher's Signature..........

2021.06.11 17:00

Page No.
Date

$= {}^{10}C_r \cdot 2^{10-r} (-7)^r \cdot x^{20-2r}$

$\Rightarrow {}^{10}C_r \cdot 2^{10-r} (-7)^r \cdot x^{20-2r}$

* Multinomial theorem :-

In expansion of $(x_1 + x_2 + \dots + x_m)^n$ the term involving $x_1^{p_1} \cdot x_2^{p_2} \cdot x_3^{p_3} \dots x_m^{p_m}$

$$\Rightarrow \frac{\lfloor n}{\lfloor p_1 \cdot \lfloor p_2 \cdot \lfloor p_3 \dots \lfloor p_m} \cdot x^{p_1} \cdot x^{p_2} \cdot x^{p_3} \dots x^{p_m}$$

where

$$p_1 + p_2 + p_3 - - - p_m = n$$

Q. In expansion of $(x+y+z)^9$, find coeff. of $x^2 \cdot y^3 \cdot z^4$.

$$\Rightarrow \left[\frac{\lfloor 9}{\lfloor 2 \lfloor 3 \lfloor 4} x^2 y^3 z^4 \right]$$

$$((x+y)+z)^9$$

Page No.
Date

$\Rightarrow {}^9C_0 (x+y)^9 + {}^9C_1 (x+y)^8 \cdot z + {}^9C_2 (x+y)^7 z^2$
$+ {}^9C_3 (x+y)^6 \cdot z^3 + {}^9C_4 (x+y)^5 \cdot z^4$
$+ \_\_\_\_\_\_\_\_\_$

$\Rightarrow - - - + {}^9C_4 (x+y)^5 \cdot z^4 + - - - - - -$

$\Rightarrow - - - - - + {}^9C_4 z^4 ({}^5C_0 x^5 + {}^5C_1 x^4 y - - -$
$+ {}^5C_3 x^2 y^3 - - + {}^5C_5 y^5) - -$

Coeff of $x^2 y^3 z^4 = {}^9C_4 \times {}^5C_3$

$\Rightarrow \frac{\lfloor 9}{\cancel{\lfloor 5} \lfloor 4} \times \frac{\cancel{\lfloor 5}}{\lfloor 3 \lfloor 2}$

$\Rightarrow \frac{\lfloor 9}{\lfloor 2 \lfloor 3 \lfloor 4}$

Q. In expansion of $(ax + by - cz + dw)^{10}$
find coeff of $x^2 y^2 z^3 w^3$.

$\frac{\lfloor 10}{\lfloor 2 \lfloor 2 \lfloor 3 \lfloor 3} (ax)^2 (by)^2 (-cz)^3 (dw)^3$

$\Rightarrow \frac{\lfloor 10}{\lfloor 2 \lfloor 2 \lfloor 3 \lfloor 3} a^2 b^2 (-c^3)(d^3)(x^2 y^2 z^3 w^3)$

Teacher's Signature............

2021.06.11 17:00

Page No.
Date

Q. $\left(2+x-\frac{3}{x}\right)^7$, find coeff of x^2

$\Rightarrow$ Term $= \frac{\lfloor 7}{\lfloor p_1 \lfloor p_2 \lfloor p_3} (2)^{p_1} (x)^{p_2} \left(-\frac{3}{x}\right)^{p_3}$

where $p_1+p_2+p_3=7$

$\Rightarrow \frac{\lfloor 7}{\lfloor p_1 \lfloor p_2 \lfloor p_3} \cdot 2^{p_1}(-3)^{p_3} \cdot x^{p_2-p_3}$

<u>for x^2</u>

$p_2-p_3=2$ $\quad (p_1+p_2+p_3=7)$

p_1	p_2	p_3	
5	2	0	$\frac{\lfloor 7}{\lfloor 5 \lfloor 2} \cdot 2^5 (-3)^0$
3	3	1	$\frac{\lfloor 7}{\lfloor 3 \lfloor 3 \lfloor 1} \cdot 2^3 \cdot (-3)^1$
1	4	2	$\frac{\lfloor 7}{\lfloor 1 \lfloor 4 \lfloor 2} \cdot 2^1 (-3)^2$

Sum =

Q. find Coefficient of x^5 in $(1+3x+3x^2+x^3)^6$

$(1+3x+3x^2+x^3)^6$

$$~~\text{Term}~~ = \left((1+x)^3\right)^6$$

$$= (1+x)^{18}$$

Coeff. of $x^5 = {}^{18}C_5$

* Remainder Based theorem

Q. $9^{10} \div 8$

$$9^{10} = (8+1)^{10}$$

$$\Rightarrow {}^{10}C_0 \cdot 8^{10} + {}^{10}C_1 \cdot 8^9 + {}^{10}C_2\, 8^8$$

$$- - - + {}^{10}C_9\, 8 + {}^{10}C_{10}$$

$$9^{10} \Rightarrow 8(k) + 1$$

Remainder $= 1$

Q. $7^{50} \div 8$

$$7^{50} = (8-1)^{50}$$

$$\Rightarrow {}^{50}C_0 \cdot 8^{50} - {}^{50}C_1 \cdot 8^{49}$$

$$+ {}^{50}C_2 \cdot 8^{48} - - - - - -$$

2021.06.11 17:00

$7^{50} = 8(k) + 1$

Remainder = 1

Q. $7^{39} \div 8$

$\Rightarrow 7^{39} = (8-1)^{39}$

$\Rightarrow {}^{39}C_0\, 8^{39} - {}^{39}C_1\, 8^{38} + \cdots$

$\cdots + {}^{39}C_{38}\, 8 - 1$

$\Rightarrow 8k - 1$

$\Rightarrow 8k - 8 + 7 = 8k' + 7$

Remainder = 7

Q. $17^{17} \% 8$

$17^{17} = (16+1)^{17}$

$(16+1)^{17} = {}^{17}C_0 (16)^{17} + {}^{17}C_1 (16)^{16} + \cdots$

$\cdots {}^{17}C_{16} (16) + {}^{17}C_{17}$

$(17)^{17} = {}^{17}C_0 (16)^{17} + {}^{17}C_1 (16)^{16} + \cdots$

$\cdots - {}^{17}C_{16} (16) + 1$

$\Rightarrow 16(k) + 1$

2021.06.11 17:00

$$(17)^{17} = 16k+1$$

Remainder = 1

Q. P.T $11^n - 10n - 1$ is divisible by 100 $\forall\ n \in N$

$\Rightarrow (10+1)^n - 10n - 1$

$\Rightarrow \left(^nC_0 + {^nC_1}\cdot 10 + {^nC_2}\cdot 10^2 \cdots {^nC_n}\, 10^n\right) - 10n + 1$

$\Rightarrow 10^2\left[{^nC_2} + {^nC_3}\cdot 10 + \cdots\cdots\right]$

$\Rightarrow 100k$ hence divisible by 100.

Q. find remainder when 5^{36} % 13.

$(5)^{36} \div 13$

$\Rightarrow (5^2)^{36} = (25)^{18}$

$\Rightarrow (26-1)^{18}$

$\Rightarrow {^{18}C_0}(26)^{18} - {^{18}C_1}(26)^{17} \cdots\cdots + 1$

$\Rightarrow 26k + 1$

Remainder = 1

Page No.
Date

Q. find remainder when $2^{99} \div 9$

$\Rightarrow (8)^{33} =$

$(9-1)^{33} = 9$

$\Rightarrow {}^{33}C_0 (9)^{33} - {}^{33}C_1 (9)^{32} + \cdots$

$\cdots + {}^{33}C_{32} (9) - 1$

$(8)^{33} \Rightarrow 9(K) - 1$

$(8)^{33} = 9K\ (8-9)$

$\Rightarrow 9K' + 8$

$\Rightarrow 9K' + 8$

Remainder

Q// find remainder when $2^{98} \% 9$

$\Rightarrow (2)^{98} \div 9$

$(2^2)^{96} =$

$2^2, 2^{96}$

$\Rightarrow 4\left[(2^3)^{32}\right]$

$\Rightarrow 4\left[(9-1)^{32}\right]$

$\Rightarrow 4\left[9K+1\right]$

Teacher's Signature

2021.06.11 17:00

Page No.
Date

$\Rightarrow 9k + 4$

Remainder $= 4$

Q. find remainder when $7^{100} \div 100$.

$\Rightarrow 7^3 \cdot 7^{98}$

$7^{100} = (7^4)^{25}$

$\Rightarrow (2401)^{25}$

$\Rightarrow (2400 + 1)^{25}$

$\Rightarrow 2400k + 1$

Remainder $= 1$

Note (i) $\Rightarrow a^n + b^n$ is always divisible by $(a+b)$ if n is odd natural number.

(ii) $a^n - b^n$ is always divisible by $(a-b)$ for all natural No. f

(iii) $a^n - b^n$ is always divisible by $a+b$ if n is even natural number.

Teacher's Signature..........

2021.06.11 17:00

Page No.
Date

★ Integral part & fractional part based problem.

Q. Prove that integral part of $(2+\sqrt{3})^n$ is always odd for all $n \in$ natural no.

$$I + f = (2+\sqrt{3})^n = \left[{}^nC_0 \cdot 2^n + {}^nC_1 \cdot 2^{n-1}(\sqrt{3}) + \cdots\cdots \right]$$

$$f' = (2-\sqrt{3})^n = \left[{}^nC_0 \cdot 2^n - {}^nC_1 \cdot 2^{n-1}(\sqrt{3}) + {}^nC_2 \cdot 2^{n-2}(\sqrt{3})^2 + \cdots \right]$$

$$\Rightarrow I + f + f' = 2\left[{}^nC_0 \cdot 2^n + {}^nC_2 \cdot 2^{n-2}(\sqrt{3})^2 + {}^nC_4 \cdot 2^{n-4}(\sqrt{3})^4 - \cdots \right]$$

$I + f + f' = 2k =$ even integer

$\Rightarrow f + f'$ is also an integer

$0 < f < 1$

$0 < f' < 1$

$\Rightarrow 0 < f + f' < 2$

Teacher's Signature..........

2021.06.11 17:00

Scanned with CamScanner

Page No.
Date

$\boxed{f + f' = 1}$ — (iii)

using (i) & (iii)

$I + 1 = 2k$

$I = (2k - 1) =$ odd integer

Q. find integral part of $(2+\sqrt{3})^6$

$\Rightarrow I + f = (2+\sqrt{3})^6 = \left[{}^6C_0 \cdot (2)^6 + {}^6C_1 (2)^5 (\sqrt{3}) + {}^6C_2 (2)^4 (\sqrt{3})^2 + {}^6C_3 (2)^3 (\sqrt{3})^3 - - - \right]$

$0 + f' = (2-\sqrt{3})^6 = \left[{}^6C_0 (2)^6 - {}^6C_1 (2)^5 (\sqrt{3}) + {}^6C_2 (2)^4 (\sqrt{3})^2 - {}^6C_3 (2)^3 (\sqrt{3})^3 + - - - \right]$

$I + f + f' = 2\left[{}^6C_0 (2)^6 + {}^6C_2 (2)^4 (\sqrt{3})^2 + {}^6C_4 (2)^2 (\sqrt{3})^4 + {}^6C_6 (\sqrt{3})^6 \right]$

$I + f + f' = 2\left[64 + 1440 + 540 + 27 \right]$

$\Rightarrow 2\left[\cancel{2071} \right]$ $\quad 2 \times (1351)$

$\Rightarrow \cancel{4142}$ $\quad = 2702$

$f + f' = 1$

Teacher's Signature............

Page No.
Date

$I + 1 = 2702$

$\boxed{I = 2701}$

Q. If $I + f = (2+\sqrt{3})^n$ find $(I+f)(1-f)$

(using 1st question)

$\Rightarrow \quad f + f' = 1 \quad \left[\because f' = (2-\sqrt{3})^n \right.$

$1 - f = f' = (2-\sqrt{3})^n$

$(I+f)(1-f) = (2+\sqrt{3})^n (2-\sqrt{3})^n$

$\Rightarrow (4-3)^n = 1$

Q. P.T integral part of $(3\sqrt{3}+5)^{2n+1}$ is even $\forall\ n \in N$.

$\Rightarrow \quad (3\sqrt{3}+5)^{2n+1}$

$I + f = (3\sqrt{3}+5)^{2n+1} =$

$0 + f' = (3\sqrt{3}-5)^{2n+1} =$

(1) $\Rightarrow \quad {}^{2n+1}C_0 (3\sqrt{3})^{2n+1} + {}^{2n+1}C_1 (3\sqrt{3})^{2n} (5)$

${}^{2n+1}C_2 (3\sqrt{3})^{2n-1} (5)^2 + - -$

Teacher's Signature...........

Page No.
Date

$$= \left[{}^{2n+1}C_0 (3\sqrt{3})^{2n+1} - {}^{2n+1}C_1 (3\sqrt{3})^{2n} \cdot 5 + {}^{2n+1}C_2 \cdot (3\sqrt{3})^{2n-1} 5^2 + \dots \right)$$

substract $\Rightarrow$

$$I + f - f' = 2\left[{}^{2n+1}C_1 (3\sqrt{3})^{2n} \cdot 5 + {}^{2n+1}C_3 (3\sqrt{3})^{2n-2} \cdot 5^3 - - - - \right]$$

$I + f - f' = 2k$ [even integer] — (i)

$\Rightarrow f - f'$ is also an integer.

$$0 < f < 1$$
$$0 < f' < 1$$

$$-1 < f - f' < 1$$

$$f - f' = 0 \text{ — (ii)}$$

$$\boxed{f = f'}$$

using (i) & (ii)

$I = 2k =$ even integer

Q. If $(3\sqrt{3}+5)^{7} = I + f$ find $(I+f)f$.

2021.06.11 17:00

$\cancel{I+f} = f'$ $f' = f$ $\left[\because f = (3\sqrt{3}-5)^7\right]$

$(I+f)f = (3\sqrt{3}+5)^7 \cdot f'$

$\Rightarrow (3\sqrt{3}+5)^7 (3\sqrt{3}-5)$

$\Rightarrow 2^7 = 128$

☆ <u>Summation of Series:-></u>

Type:-1 $\sum_{r=0}^{n} {}^nC_r \cdot (x)^{n-r} \cdot (y)^r = (x+y)^n$

Q. ${}^nC_0 (8^1) + {}^nC_1 \cdot 8^2 + {}^nC_2 \cdot 8^3 + \cdots$

$\cdots {}^nC_n\, 8^{n+1}$

$\Rightarrow \cancel{(8+1)^n} \quad 8[8+1]^n$

$\Rightarrow 8[9^n]$

Q. 2 $10_{C_0} 9^0 + 10_{C_1} \cdot 9^1 + 10_{C_2} 9^2 + 10_{C_3} \cdot 9^3 \cdots$

$\cdots + 10_{C_{10}} 9^{10}$

$\Rightarrow \cancel{1+(10)^{10}}$

$\Rightarrow (10)^{10} - 1$