Q] Consider a triangle ABC with side lengths $a = 4\sqrt{5}$, $b = 4$, $c = 8$ and side BC lies along the line $x + 2y - 8 = 0$. Let the line joining the vertex A and the incentre of the triangle (I) be L_1. If L_1 cuts the side BC at $S\left(\frac{8}{3}, \frac{8}{3}\right)$. Find out the Following -:

(a) Coordinates of B is $(b_1 = ?, b_2 = ?)$

(b) Coordinates of C is $(c_1 = ?, c_2 = ?)$

(c) Coordinates of A is $(a_1 = ?, a_2 = ?)$

(d) Coordinates of I is $(i_1 = ?, i_2 = ?)$

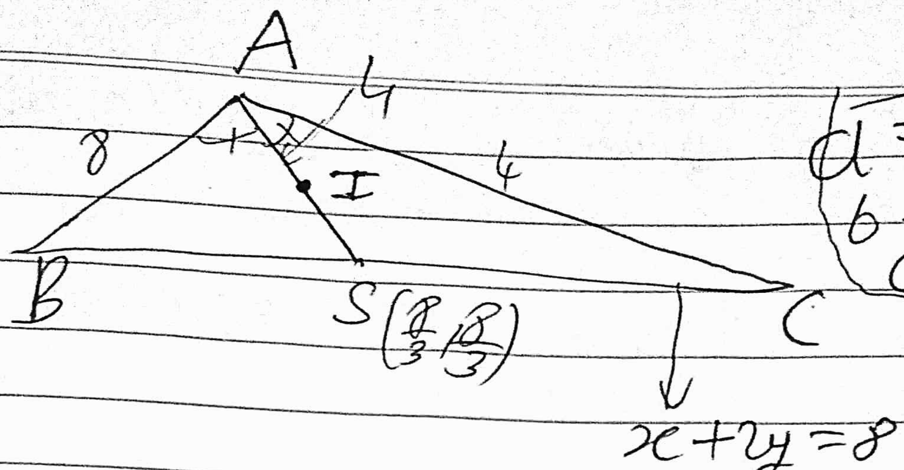
Also

Given: $a_1 + a_2 \leq 0$

&

$b_1 + b_2 \geq 8$

$\boxed{\text{Sol}^n}$



$$\boxed{\begin{aligned} a &= 4\sqrt{5} \\ b &= 4 \\ c &= 8 \end{aligned}}$$ ☆

We know AS or L_1 is the angle bisector of A. [Property of Incentre]

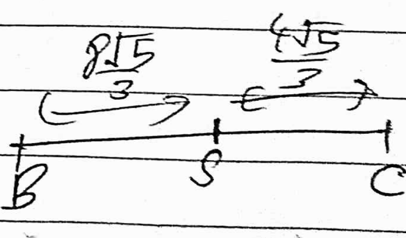
We say that, $\dfrac{AB}{AC} = \dfrac{SB}{SC}$ [Property of $\angle$ bisector]

& $SB + SC = BC$

$$\frac{SB}{SC} = \frac{c}{b} = \frac{2}{1}$$

$$SB = \left(\frac{2}{3}BC\right) \text{ and } SC = \left(\frac{1}{3}(BC)\right)$$

$$\boxed{\begin{aligned} SB &= \frac{2}{3}a = \frac{8\sqrt{5}}{3} \\ SC &= \frac{a}{3} = \frac{4\sqrt{5}}{3} \end{aligned}}$$



To use parametric form to find Coordinates of B & C.

Line $x+2y=8$

$$\tan\theta = -\frac{1}{2} \Rightarrow \cos\theta = \frac{-2}{\sqrt{5}} \;\Big|\; \sin\theta = \frac{1}{\sqrt{5}}$$

$$B \equiv \left(\frac{8}{3} \pm (SB)\cos\theta , \ \frac{8}{3} \pm (SB)\sin\theta \right)$$

$$C \equiv \left(\frac{8}{3} \mp (SC)\cos\theta , \ \frac{8}{3} \mp (SC)\sin\theta \right)$$

$$B \equiv \left(\frac{8}{3} \pm \frac{8\sqrt{5}}{3}\left(\frac{-2}{\sqrt{5}}\right) , \ \frac{8}{3} \pm \left(\frac{8\sqrt{5}}{3}\right)\left(\frac{1}{\sqrt{5}}\right) \right)$$

$$C \equiv \left(\frac{8}{3} \mp \left(\frac{4\sqrt{5}}{3}\right)\left(\frac{-2}{\sqrt{5}}\right) , \ \frac{8}{3} \mp \left(\frac{4\sqrt{5}}{3}\right)\left(\frac{1}{\sqrt{5}}\right) \right)$$

2 cases

$$\left[B = (8,0) \ \& \ C = (0,4) \right] \text{ — } 1$$

or

$$\left[B = \left(\frac{-8}{3}, \frac{16}{3}\right) \ \& \ C = \left(\frac{16}{3}, \frac{4}{3}\right) \right] \text{ — } 2$$

Since $b_1 + b_2 \geq 8$

We take case 1 $(0+8 \geq 8)$

$$\boxed{B = (8,0) \ \& \ C = (0,4)}$$

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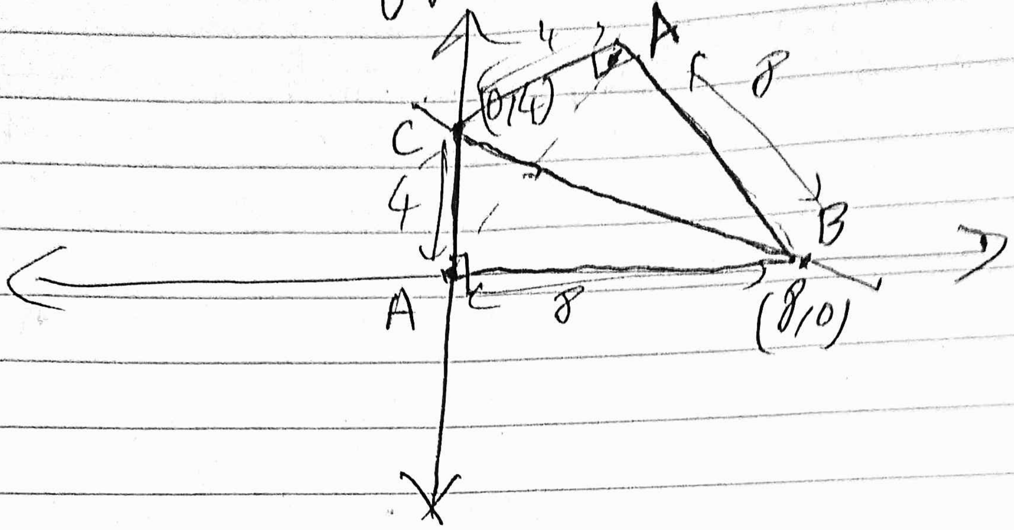
(C)

To find $A(a_1, a_2)$

we draw figure



from the figure, there are

2 possible ~~values for~~ points for A.

one is obviously the origin.

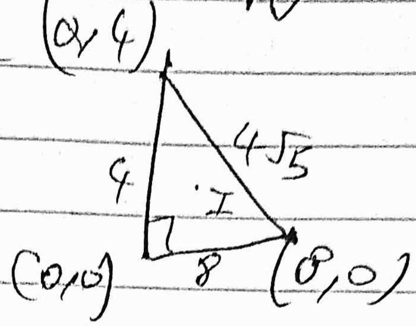
$A = (0,0)$ $[a_1 + a_2 \leq 0]$

other is in first quadrant (~~x+y>0~~)

for that point $a_1 + a_2 > 0$

$\boxed{\text{Hence the required A is } (0,0)}$

(d) Co-ordinates of I is simply calculated through formula



$$I = \left(\frac{a x_1 + b x_2 + c x_3}{a+b+c} , \frac{a y_1 + b y_2 + c y_3}{a+b+c} \right)$$

$$I = \left(\frac{8(4) + 0 + 0}{4+8+4\sqrt{5}} , \frac{4(8) + 0 + 0}{4+8+4\sqrt{5}} \right)$$

$$\boxed{I = \left(\frac{8}{3+\sqrt{5}} , \frac{8}{3+\sqrt{5}} \right)} = \left(2(3-\sqrt{5}) , 2(3-\sqrt{5}) \right)$$