Rotation Trick

If Line $L: ax+by+c=0$ is to be rotated by angle θ about point (x_1, y_1)

Just rotate the vector $a+bi$ by θ. [ie multiply by $e^{i\theta}$]

$$(a+bi)e^{i\theta} = (a\cos\theta - b\sin\theta) + (b\cos\theta + a\sin\theta)i$$
$$= a' + b'i \quad \text{[say]}$$

eqn of rotated line is

$$a'(x-x_1) + b'(y-y_1) = 0$$

or

$$\boxed{a'x + b'y = a'x_1 + b'y_1}$$

Example ⇒ Rotate $2x+y=3$ about $(1,1)$ by $45°$ anticlockwise.

Soln $(2+i)(\sin 45 i + \cos 45) = \frac{1}{\sqrt{2}}(2+i)(i+1)$

$$= \frac{3i+1}{\sqrt{2}} \equiv (1,3)$$

eqn of new line

$$1\cdot(x-1) + 3(y-1) = 0$$

or

$$x+3y = 1\cdot 1 + 3\cdot 1 = 4$$

$$\boxed{x+3y=4}$$