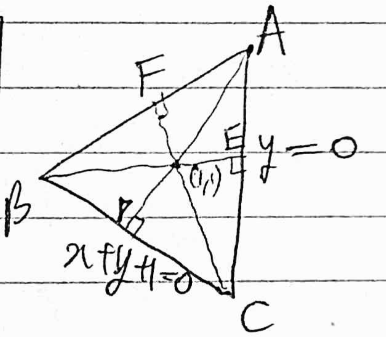
Q1 Consider a triangle whose 2 sides lie on x-axis and the line $x+y+1=0$. If the orthocentre of the triangle is $(1,1)$. Then

[A] Circumcentre of the triangle is $(\_,\_)$
[B] Centroid of triangle is $(\_,\_)$
[C] Radius of circumcircle is ____
[D] LENGTH of median passing through $(0,0)$ Vertex is ____

Soln



C is intersection of $y=0$ & $x+y+1=0$

i.e. $C \equiv (-1,0)$

Altitude from A is $\perp$ to BC

m_{AD} = slope of altitude $= \frac{-1}{m_{BC}} = \frac{-1}{(-1)}$

m_{AD} = slope of altitude from A is 1

Similarly slope of altitude from B $= \frac{-1}{m_{AC}} = \infty$ (infinity)

$(m_{AE}) = 0$

eqn of AD $= \boxed{(y-1) = 1\cdot(x-1)}$

$= \boxed{y-x=0}$ — Eqn of AD

eqn of BE $= \boxed{0(y-1) = \cancel{\ }(x-1)}$

$\Rightarrow x \boxed{\cancel{y} -1=0}$ — Eqn of BE

A is intersection of line AD & AC

ie $\left[\begin{array}{l} y-x=0 \\ y=0 \end{array}\right. \Rightarrow A \equiv (0,0)$

B is intersection of line BC & BE

ie $\left[\begin{array}{l} x+y+1=0 \\ x-1=0 \end{array}\right. \Rightarrow B \equiv \cancel{(2,1)}$

$(1,-2)$

Hence all co-ordinates of vertices are

$\left.\begin{array}{l} A \equiv (0,0) \\ B \equiv (1,-2) \\ C \equiv (-1,0) \end{array}\right] \quad G \equiv \left(\frac{0+1-1}{3}, \frac{0-2+0}{3}\right)$

Centroid $\equiv \left(0, \frac{-2}{3}\right) \equiv G\left(0, -\frac{2}{3}\right)$ [B]

Now,

$\underset{C}{|} \overset{1}{\text{———}} \underset{G}{|} \overset{2}{\text{———}} \underset{O}{|}$

Circumcentre.

$\boxed{\vec{C} = \frac{3\vec{G} - \vec{O}}{2}}$

$\cancel{C \equiv \left(\frac{-3-1}{2}, \frac{1-1}{2}\right)}$

$\cancel{C \equiv (-2, 0)}$

$C = \left(\frac{0-1}{2}, \frac{-2-1}{2}\right)$

$C = \left(-\frac{1}{2}, -\frac{3}{2}\right)$

[A]

Radius of circumcircle is distance between any vertex & circumcentre.

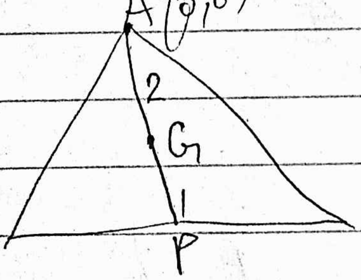
dist b/w $(0,0)$ and $\left(-\frac{1}{2}, -\frac{3}{2}\right) = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

$= \sqrt{\frac{10}{4}} = \boxed{\sqrt{\frac{5}{2}}}$ [C]

[Alternate Method : $R = \frac{a}{2\sin A} = \frac{\sqrt{5}}{2\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{\frac{5}{2}}$]

[D]

Length of median passing through $(0,0)$ ie A



$AP = ?$

We know $\frac{AG}{GP} = \frac{2}{1}$ (Prop

$AP = \frac{3}{2}(AG)$

$AP = \frac{3}{2}\left(\sqrt{(0-0)^2 + \left(\frac{-2}{3}-0\right)^2}\right)$

$= \left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$

Length of median AP $= 1$ [D]