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Q2]

Q. Vertices of a variable triangle are $(3,4)$, $(5\cos\theta, 5\sin\theta)$, $(5\sin\theta, -5\cos\theta)$ where $\theta \in [0, 2\pi]$. Locus of its orthocentre is

(a) $x^2+y^2+6x+8y-25=0$

(b) $x^2+y^2-6x+8y-25=0$

(c) $x^2+y^2+6x-8y-25=0$

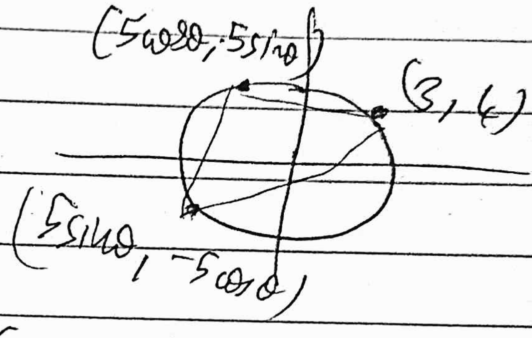
(d) $x^2+y^2-6x-8y-25=0$

Note $\Rightarrow$

Sol^n Traditional method is too long
(finding eq^n of altitudes & intersection)

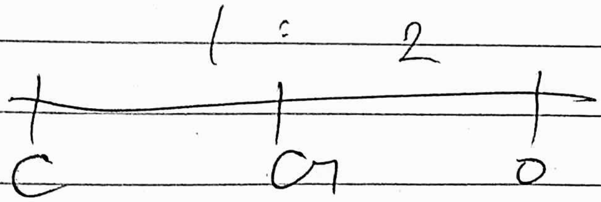
T Short method $\Rightarrow$

Observe all 3 points lie on a circle
$x^2+y^2 = 5^2$



Hence circumcentre is $(0,0)$

Centroid is $G \equiv \left(\dfrac{3+5\cos\theta+5\sin\theta}{3}, \dfrac{4+5\sin\theta-5\cos\theta}{3}\right)$



$$\vec{O} = 3\vec{G} - 2\vec{C}$$

$$O = \left(3+5\cos\theta+5\sin\theta - 0, 4+5\sin\theta-5\cos\theta - 0\right)$$

$$h = 3+5(\cos\theta+\sin\theta)$$
$$k = 4+5(\sin\theta-\cos\theta)$$

$$(h-3)^2+(k-4)^2 = 25\left((\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2\right)$$
$$(h-3)^2+(k-4)^2 = 50$$
$$h^2+k^2-6h-8k-25=0 \quad (d) \text{ Ans}$$