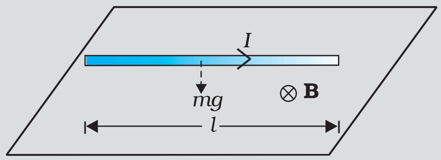


FIGURE 4.3

Solution From Eq. (4.4), we find that there is an upward force **F**, of magnitude IlB,. For mid-air suspension, this must be balanced by the force due to gravity:

$$m\,g = I\,lB$$

$$B = \frac{m\,g}{I\,l}$$

$$= \frac{0.2 \cdot 9.8}{2 \cdot 1.5} = 0.65 \text{ T}$$

Note that it would have been sufficient to specify m/l, the mass per unit length of the wire. The earth's magnetic field is approximately 4×10^{-5} T and we have ignored it.

EXAMPLE 4.1

Example 4.2 If the magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis (Fig. 4.4), which way would the Lorentz force be for (a) an electron (negative charge), (b) a proton (positive charge).

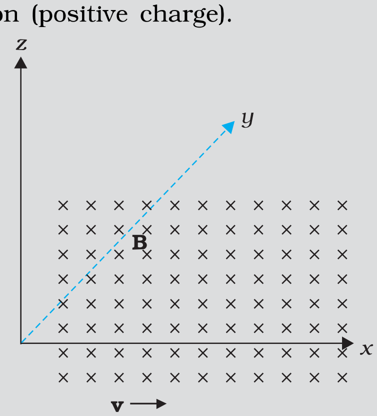


FIGURE 4.4

Solution The velocity **v** of particle is along the x-axis, while **B**, the magnetic field is along the y-axis, so **v × B** is along the z-axis (screw rule or right-hand thumb rule). So, (a) for electron it will be along $-z$ axis. (b) for a positive charge (proton) the force is along $+z$ axis.

EXAMPLE 4.2

4.3 MOTION IN A MAGNETIC FIELD

We will now consider, in greater detail, the motion of a charge moving in a magnetic field. We have learnt in Mechanics (see Class XI book, Chapter 6) that a force on a particle does work if the force has a component along (or opposed to) the direction of motion of the particle. In the case of motion

PHYSICS

Charged particles moving in a magnetic field. Interactive demonstration:
http://www.phys.hawaii.edu/~teb/optics/java/partmagn/index.html

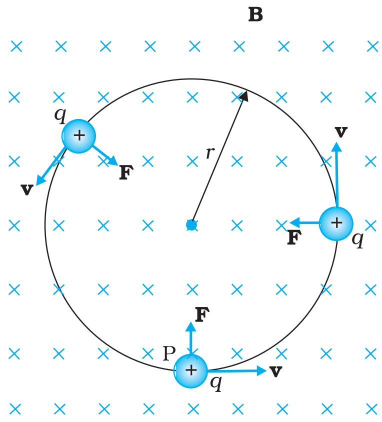


FIGURE 4.5 Circular motion

of a charge in a magnetic field, the magnetic force is perpendicular to the velocity of the particle. So no work is done and no change in the magnitude of the velocity is produced (though the direction of momentum may be changed). [Notice that this is unlike the force due to an electric field, $q\mathbf{E}$, which *can* have a component parallel (or antiparallel) to motion and thus can transfer energy in addition to momentum.]

We shall consider motion of a charged particle in a *uniform* magnetic field. First consider the case of $\mathbf{v}$ perpendicular to $\mathbf{B}$. The perpendicular force, $q\,\mathbf{v} \times \mathbf{B}$, acts as a centripetal force and produces a circular motion perpendicular to the magnetic field. *The particle will describe a circle if* $\mathbf{v}$ *and* $\mathbf{B}$ *are perpendicular to each other* (Fig. 4.5).

If velocity has a component along $\mathbf{B}$, this component remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. The motion in a plane perpendicular to $\mathbf{B}$ is as before a circular one, thereby producing a *helical motion* (Fig. 4.6).

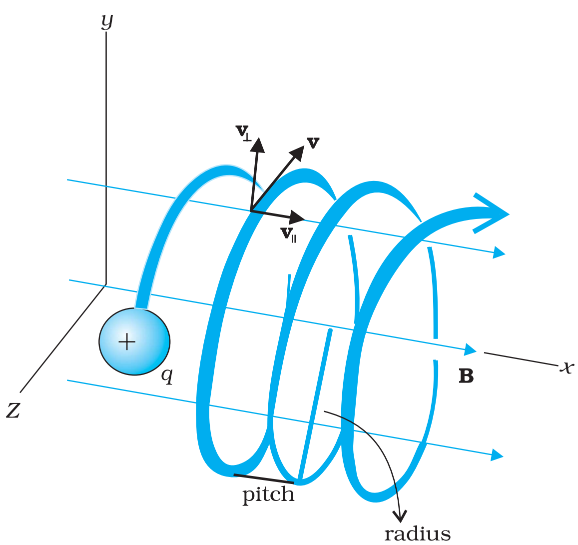


FIGURE 4.6 Helical motion

You have already learnt in earlier classes (See Class XI, Chapter 4) that if r is the radius of the circular path of a particle, then a force of $m\,v^2 / r$, acts perpendicular to the path towards the centre of the circle, and is called the centripetal force. If the velocity $\mathbf{v}$ is perpendicular to the magnetic field $\mathbf{B}$, the magnetic force is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ and acts like a centripetal force. It has a magnitude $q\,v\,B$. Equating the two expressions for centripetal force,

$m\,v^2/r = q\,v\,B$, which gives

$$r = m\,v \,/\, qB \qquad (4.5)$$

for the radius of the circle described by the charged particle. The larger the momentum, the larger is the radius and bigger the circle described. If ω is the angular frequency, then $v = \omega\, r$. So,

$$\omega = 2\pi\,\nu = q\,B/\,m \qquad [4.6(a)]$$

which is independent of the velocity or energy . Here ν is the frequency of rotation. The independence of ν from energy has important application in the design of a cyclotron (see Section 4.4.2).

The time taken for one revolution is $T = 2\pi/\omega \equiv 1/\nu$. If there is a component of the velocity parallel to the magnetic field (denoted by $v_{||}$), it will make the particle move along the field and the path of the particle would be a helical one (Fig. 4.6). The distance moved along the magnetic field in one rotation is called pitch p. Using Eq. [4.6 (a)], we have

$$p = v_{||}T = 2\pi m\,v_{||} \,/\, q\,B \qquad [4.6(b)]$$

The radius of the circular component of motion is called the *radius* of the *helix*.

Example 4.3 What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3×10^{7} m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV. (1 eV = 1.6×10^{-19} J).

Solution Using Eq. (4.5) we find

$r = mv / (qB) = 9 \times 10^{-31}\text{ kg} \times 3 \times 10^{7}\text{ m s}^{-1} / (1.6 \times 10^{-19}\text{ C} \times 6 \times 10^{-4}\text{ T})$

$= 26 \times 10^{-2}\text{ m} = 26\text{ cm}$

$\nu = v / (2\pi r) = 2 \times 10^{6}\text{ s}^{-1} = 2 \times 10^{6}\text{ Hz} = 2\text{ MHz}.$

$E = (½)mv^{2} = (½)\, 9 \times 10^{-31}\text{ kg} \times 9 \times 10^{14}\text{ m}^2/\text{s}^2 = 40.5 \times 10^{-17}\text{ J}$

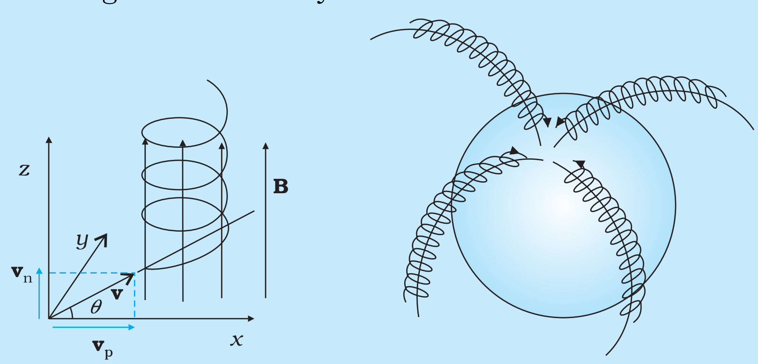
$\approx 4 \times 10^{-16}\text{ J} = 2.5\text{ keV}.$

EXAMPLE 4.3

HELICAL MOTION OF CHARGED PARTICLES AND AURORA BORIOLIS

In polar regions like Alaska and Northern Canada, a splendid display of colours is seen in the sky. The appearance of dancing green pink lights is fascinating, and equally puzzling. An explanation of this natural phenomenon is now found in physics, in terms of what we have studied here.

Consider a charged particle of mass *m* and charge *q*, entering a region of magnetic field **B** with an initial velocity **v**. Let this velocity have a component $\mathbf{v}_p$ parallel to the magnetic field and a component $\mathbf{v}_n$ normal to it. There is no force on a charged particle in the direction of the field. Hence the particle continues to travel with the velocity $\mathbf{v}_p$ parallel to the field. The normal component $\mathbf{v}_n$ of the particle results in a Lorentz force ($\mathbf{v}_n \times \mathbf{B}$) which is perpendicular to both $\mathbf{v}_n$ and **B**. As seen in Section 4.3.1 the particle thus has a tendency to perform a circular motion in a plane perpendicular to the magnetic field. When this is coupled with the velocity parallel to the field, the resulting trajectory will be a helix along the magnetic field line, as shown in Figure (a) here. Even if the field line bends, the helically moving particle is trapped and guided to move around the field line. Since the Lorentz force is normal to the velocity of each point, the field does no work on the particle and the magnitude of velocity remains the same.



(a) (b)

During a solar flare, a large number of electrons and protons are ejected from the sun. Some of them get trapped in the earth's magnetic field and move in helical paths along the field lines. The field lines come closer to each other near the magnetic poles; see figure (b). Hence the density of charges increases near the poles. These particles collide with atoms and molecules of the atmosphere. Excited oxygen atoms emit green light and excited nitrogen atoms emits pink light. This phenomenon is called *Aurora Boriolis* in physics.

4.4 Motion in Combined Electric and Magnetic Fields

4.4.1 Velocity selector

You know that a charge q moving with velocity $\mathbf{v}$ in presence of both electric and magnetic fields experiences a force given by Eq. (4.3), that is,

$$\mathbf{F} = q\,(\mathbf{E} + \boldsymbol{v} \times \mathbf{B}) = \mathbf{F}_E + \mathbf{F}_B$$

We shall consider the simple case in which electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of the particle, as shown in Fig. 4.7. We have,

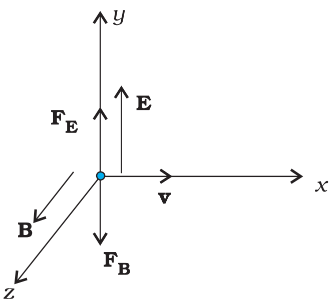


FIGURE 4.7

$$\mathbf{E} = E\,\hat{\mathbf{j}},\ \mathbf{B} = B\,\hat{\mathbf{k}},\ \mathbf{v} = v\,\hat{\mathbf{i}}$$

$$\mathbf{F}_E = q\mathbf{E} = qE\,\hat{\mathbf{j}},\ \mathbf{F}_B = q\mathbf{v} \times \mathbf{B},\ = q\left(v\,\hat{\mathbf{i}} \times B\hat{\mathbf{k}}\right) = -qB\,\hat{\mathbf{j}}$$

Therefore, $\mathbf{F} = q\left(E - vB\right)\hat{\mathbf{j}}$.

Thus, electric and magnetic forces are in opposite directions as shown in the figure. Suppose, we adjust the value of **E** and **B** such that magnitude of the two forces are equal. Then, total force on the charge is zero and the charge will move in the fields undeflected. This happens when,

$$qE = qvB \quad \text{or} \quad v = \frac{E}{B} \tag{4.7}$$

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass). The crossed E and B fields, therefore, serve as a *velocity selector*. Only particles with speed E/B pass undeflected through the region of crossed fields. This method was employed by J. J. Thomson in 1897 to measure the charge to mass ratio (e/m) of an electron. The principle is also employed in Mass Spectrometer – a device that separates charged particles, usually ions, according to their charge to mass ratio.

Cyclotron
Interactive demonstration:
http://www.phy.ntnu.edu.tw/ntnujava/viewtopic.php?t=50

4.4.2 Cyclotron

The cyclotron is a machine to accelerate charged particles or ions to high energies. It was invented by E.O. Lawrence and M.S. Livingston in 1934 to investigate nuclear structure. The cyclotron uses both electric and magnetic fields in combination to increase the energy of charged particles. As the fields are perpendicular to each other they are called *crossed fields*. Cyclotron uses the fact that the frequency of revolution of the charged particle in a magnetic field is independent of its energy. The particles move most of the time inside two semicircular disc-like metal containers, D_1 and D_2, which are called *dees* as they look like the letter D. Figure 4.8 shows a schematic view of the cyclotron. Inside the metal boxes the particle is shielded and is not acted on by the electric field. The magnetic field, however, acts on the particle and makes it go round in a circular path inside a dee. Every time the particle moves from one dee to another it is acted upon by the electric field. The sign of the electric field is changed alternately in tune with the circular motion of the particle. This ensures that the particle is always accelerated by the electric field. Each time the acceleration increases the energy of the particle. As energy