Differential Equations

- Order of a differential equation is the order of the highest order derivative occuring in the differential equation.
- Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.

> Methods of solving a first order first degree differential equation:

(a) Differential equation of the form $\frac{dy}{dx} = f(x)$

$$\frac{dy}{dx} = f(x) \Rightarrow dy = f(x)\,dx$$

Integrating both sides we obtain

$$\int dy = \int f(x)\,dx + c \quad \text{or} \quad y = \int f(x)\,dx + c$$

(b) Differential equation of the form $\frac{dy}{dx} = f(x)\,g(y)$

$$\frac{dy}{dx} = f(x)\,g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx + c$$

(c) Differential equation of the form of $\frac{dy}{dx} = f(ax+by+c)$:

To solve this type of differential equations, we put

$$ax + by + c = v \quad \text{and} \quad \frac{dy}{dx} = \frac{1}{b}\left(\frac{dv}{dx} - a\right)$$

$$\therefore \quad \frac{dv}{a + b f(v)} = dx$$

So solution is by integrating $\int \frac{dv}{a + b f(v)} = \int dx$

(d) Differential Equation of homogenous Type:

An equation in x and y is said to be homogenous if it can be put in the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where $f(x,y)$ and $g(x,y)$ are both homogenous functions of the same degree in x & y.

So to solve the homogenous differential equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$, substitute $y = vx$ and so $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Thus $v + x\frac{dv}{dx} = f(v) \Rightarrow \frac{dx}{x} = \frac{dv}{f(v) - v}$

Therefore solution is $\int \frac{dx}{x} = \int \frac{dv}{f(v) - v} + c$

> Linear Differential Equation.

$$\frac{dy}{dx} + \underline{Py} = Q$$

Where P and Q are either constants or functions of x.
Multiplying both sides of (i) by $e^{\int P\,dx}$, we get

$$y e^{\int P\,dx} = \int Q e^{\int P\,dx} + c$$

which is the required solution, where c is the constant and $\underline{e^{\int P\,dx}}$ is called the <u>integration factor</u>.