

An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30° with the initial velocity.

Magnetic field strength, $B = 0.15 \text{ T}$

Charge on the electron, $e = 1.6 \times 10^{-19} \text{ C}$

Mass of the electron, $m = 9.1 \times 10^{-31} \text{ kg}$

Potential difference, $V = 2.0 \text{ kV} = 2 \times 10^3 \text{ V}$

Thus, kinetic energy of the electron = eV

$$\Rightarrow eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} \quad \dots (1)$$

Where,

v = velocity of the electron

(a) Magnetic force on the electron provides the required centripetal force of the electron.

Hence, the electron moves in a circular path of radius r .

Magnetic force on the electron is given by the relation,

$B ev$

Centripetal force = $\frac{mv^2}{r}$

$$\therefore Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be} \quad \dots (2)$$

From equations (1) and (2), we get

$$\begin{aligned} r &= \frac{m}{Be} \left[\frac{2eV}{m} \right]^{\frac{1}{2}} \\ &= \frac{9.1 \times 10^{-31}}{0.15 \times 1.6 \times 10^{-19}} \times \left(\frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^3}{9.1 \times 10^{-31}} \right)^{\frac{1}{2}} \\ &= 100.55 \times 10^{-5} \\ &= 1.01 \times 10^{-3} \text{ m} \\ &= 1 \text{ mm} \end{aligned}$$

Hence, the electron has a circular trajectory of radius 1.0 mm normal to the magnetic field.

(b) When the field makes an angle θ of 30° with initial velocity, the initial velocity will be,

$$v_1 = v \sin \theta$$

From equation (2), we can write the expression for new radius as:

$$\begin{aligned} r_1 &= \frac{mv_1}{Be} \\ &= \frac{mv \sin \theta}{Be} \\ &= \frac{9.1 \times 10^{-31}}{0.15 \times 1.6 \times 10^{-19}} \times \left[\frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^3}{9.1 \times 10^{-31}} \right]^{\frac{1}{2}} \times \sin 30^\circ \\ &= 0.5 \times 10^{-3} \text{ m} \\ &= 0.5 \text{ mm} \end{aligned}$$