Substituting the values of $\frac{d^2y}{dx^2}$ and y in the given differential equation, we get

L.H.S. = $(-a\cos x - b\sin x) + (a\cos x + b\sin x) = 0$ = R.H.S.

Therefore, the given function is a solution of the given differential equation.

EXERCISE 9.2

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1$: $y'' - y' = 0$

2. $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$

3. $y = \cos x + C$: $y' + \sin x = 0$

4. $y = \sqrt{1+x^2}$: $y' = \frac{xy}{1+x^2}$

5. $y = Ax$: $xy' = y \ (x \neq 0)$

6. $y = x\sin x$: $xy' = y + x\sqrt{x^2 - y^2} \ (x \neq 0 \text{ and } x > y \text{ or } x < -y)$

7. $xy = \log y + C$: $y' = \frac{y^2}{1-xy} \ (xy \neq 1)$

8. $y - \cos y = x$: $(y\sin y + \cos y + x)\, y' = y$

9. $x + y = \tan^{-1}y$: $y^2 y' + y^2 + 1 = 0$

10. $y = \sqrt{a^2 - x^2} \ x \in (-a, a)$: $x + y\frac{dy}{dx} = 0 \ (y \neq 0)$

11. The number of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4

12. The number of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

9.4 Formation of a Differential Equation whose General Solution is given

We know that the equation

$$x^2 + y^2 + 2x - 4y + 4 = 0 \quad \text{... (1)}$$

represents a circle having centre at $(-1, 2)$ and radius 1 unit.

Differentiating equation (1) with respect to x, we get

$$\frac{dy}{dx} = \frac{x+1}{2-y} \quad (y \neq 2) \qquad \text{... (2)}$$

which is a differential equation. You will find later on [See (example 9 section 9.5.1.)] that this equation represents the family of circles and one member of the family is the circle given in equation (1).

Let us consider the equation

$$x^2 + y^2 = r^2 \qquad \text{... (3)}$$

By giving different values to r, we get different members of the family e.g. $x^2 + y^2 = 1$, $x^2 + y^2 = 4$, $x^2 + y^2 = 9$ etc. (see Fig 9.1). Thus, equation (3) represents a family of concentric circles centered at the origin and having different radii.

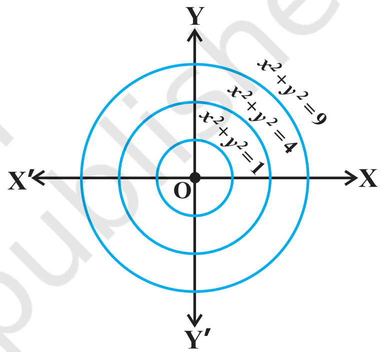

Fig 9.1

We are interested in finding a differential equation that is satisfied by each member of the family. The differential equation must be free from r because r is different for different members of the family. This equation is obtained by differentiating equation (3) with respect to x, i.e.,

$$2x + 2y\frac{dy}{dx} = 0 \quad \text{or} \quad x + y\frac{dy}{dx} = 0 \qquad \text{... (4)}$$

which represents the family of concentric circles given by equation (3).

Again, let us consider the equation

$$y = mx + c \qquad \text{... (5)}$$

By giving different values to the parameters m and c, we get different members of the family, e.g.,

$y = x$ $\quad (m = 1, \ c = 0)$

$y = \sqrt{3}\,x$ $\quad (m = \sqrt{3}, \ c = 0)$

$y = x + 1$ $\quad (m = 1, \ c = 1)$

$y = -x$ $\quad (m = -1, \ c = 0)$

$y = -x - 1$ $\quad (m = -1, \ c = -1)$ etc. (see Fig 9.2).

Thus, equation (5) represents the family of straight lines, where m, c are parameters.

We are now interested in finding a differential equation that is satisfied by each member of the family. Further, the equation must be free from m and c because m and

c are different for different members of the family. This is obtained by differentiating equation (5) with respect to x, successively we get

$$\frac{dy}{dx} = m \text{, and } \frac{d^2y}{dx^2} = 0 \qquad \text{... (6)}$$

The equation (6) represents the family of straight lines given by equation (5).

Note that equations (3) and (5) are the general solutions of equations (4) and (6) respectively.

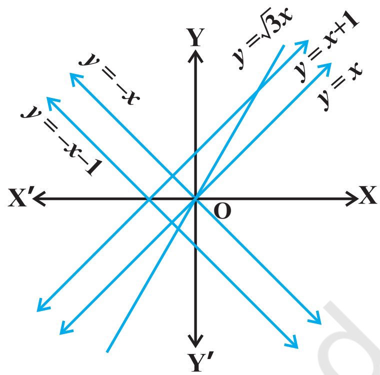

Fig 9.2

9.4.1 *Procedure to form a differential equation that will represent a given family of curves*

(a) If the given family F_1 of curves depends on only one parameter then it is represented by an equation of the form

$$F_1(x, y, a) = 0 \qquad \text{... (1)}$$

For example, the family of parabolas $y^2 = ax$ can be represented by an equation of the form $f(x, y, a) : y^2 = ax$.

Differentiating equation (1) with respect to x, we get an equation involving y', y, x, and a, i.e.,

$$g(x, y, y', a) = 0 \qquad \text{... (2)}$$

The required differential equation is then obtained by eliminating a from equations (1) and (2) as

$$F(x, y, y') = 0 \qquad \text{... (3)}$$

(b) If the given family F_2 of curves depends on the parameters a, b (say) then it is represented by an equation of the from

$$F_2(x, y, a, b) = 0 \qquad \text{... (4)}$$

Differentiating equation (4) with respect to x, we get an equation involving y', x, y, a, b, i.e.,

$$g(x, y, y', a, b) = 0 \qquad \text{... (5)}$$

But it is not possible to eliminate two parameters a and b from the two equations and so, we need a third equation. This equation is obtained by differentiating equation (5), with respect to x, to obtain a relation of the form

$$h(x, y, y', y'', a, b) = 0 \qquad \text{... (6)}$$

The required differential equation is then obtained by eliminating a and b from equations (4), (5) and (6) as

$$F(x, y, y', y'') = 0 \qquad \text{... (7)}$$

Note The order of a differential equation representing a family of curves is same as the number of arbitrary constants present in the equation corresponding to the family of curves.

Example 4 Form the differential equation representing the family of curves $y = mx$, where, m is arbitrary constant.

Solution We have

$$y = mx \qquad \text{... (1)}$$

Differentiating both sides of equation (1) with respect to x, we get

$$\frac{dy}{dx} = m$$

Substituting the value of m in equation (1) we get $y = \frac{dy}{dx} \cdot x$

or $$x\frac{dy}{dx} - y = 0$$

which is free from the parameter m and hence this is the required differential equation.

Example 5 Form the differential equation representing the family of curves $y = a \sin (x + b)$, where a, b are arbitrary constants.

Solution We have

$$y = a \sin (x + b) \qquad \text{... (1)}$$

Differentiating both sides of equation (1) with respect to x, successively we get

$$\frac{dy}{dx} = a \cos (x + b) \qquad \text{... (2)}$$

$$\frac{d^2y}{dx^2} = -a \sin (x + b) \qquad \text{... (3)}$$

Eliminating a and b from equations (1), (2) and (3), we get

$$\frac{d^2y}{dx^2} + y = 0 \qquad \text{... (4)}$$

which is free from the arbitrary constants a and b and hence this the required differential equation.