11. x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of second square with respect to the area of first square.

Sol. Given x and y are the sides of two squares such that $y = x - x^2$.

$\therefore$ Area of the first square, $A_1 = x^2$

and area of the second square, $A_2 = y^2 = (x - x^2)^2$

$$\therefore \quad \frac{dA_1}{dt} = \frac{d}{dt}(x^2) = 2x \cdot \frac{dx}{dt}$$

and
$$\frac{dA_2}{dt} = \frac{d}{dt}(x - x^2)^2$$

$$= 2(x - x^2)(1 - 2x)\frac{dx}{dt}$$

$$\therefore \quad \frac{dA_2}{dA_1} = \frac{\frac{dA_2}{dt}}{\frac{dA_1}{dt}} = \frac{(2x - 2x^2)(1 - 2x)\frac{dx}{dt}}{2x\frac{dx}{dt}}$$

$$= \frac{(1 - 2x)\,2x(1 - x)}{2x} = (1 - 2x)\,(1 - x) = 2x^2 - 3x + 1$$