2. If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.

Sol. Let the radius of circle at any time t is r.

Then area of the circle at any time t is $A = \pi r^2$

$$\therefore \quad \frac{d}{dt}A = \frac{d}{dt}(\pi r^2)$$

$$\Rightarrow \quad \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \qquad \text{(i)}$$

Since the area of a circle increases at a uniform rate, we have

$$\frac{dA}{dt} = k, \text{ where } k \text{ is a constant} \qquad \text{(ii)}$$

From (i) and (ii), we get

$$2\pi r \cdot \frac{dr}{dt} = k$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{k}{2\pi r} = \frac{k}{2\pi} \cdot \left(\frac{1}{r}\right)$$

$$\Rightarrow \quad 2\pi \frac{dr}{dt} = \frac{k}{r}$$

$$\Rightarrow \quad \frac{d(2\pi r)}{dt} = \frac{k}{r}$$

$$\Rightarrow \quad \frac{dP}{dt} = \frac{k}{r}, \text{ where } P = 2\pi r$$

$$\Rightarrow \quad \frac{dP}{dt} \propto \frac{1}{r}$$

Thus perimeter varies inversely as the radius.