(ii) $\sum \frac{2^p 3^q 4^r 5^s}{p!q!r!s!}$

$(14)^{20} = \sum 20! \frac{2^p 3^q 4^r 5^s}{p!q!r!s!}$

$\boxed{\frac{(14)^{20}}{20!}}$

(iii) $\cancel{\sum \frac{2^p \cdot 2 \; 3^q \; 3 \cdot 4 \cdot 4^r \; 5 \cdot 5^s}{p!q!r!s!}}$

$\cancel{= \frac{(14)^{20}}{20! \times 120}}$

$\sum \frac{2^p \cdot 2 \cdot \frac{9^q}{3} \cdot \frac{2^r}{2} \cdot 5^4 \left(\frac{1}{5}\right)^s}{x!}$

$= \frac{5^4}{6} \sum \frac{2^p \, 9^q \, 2^r \left(\frac{1}{5}\right)^s}{p!q!r!s!}$

$\boxed{\frac{6}{5^4} \times \frac{\left(13 + \frac{1}{5}\right)^{20}}{20!}} =$

Que: Count the Number of terms:

(A) $\sum_{r=0}^{n} T_r$

no. of terms :- $(n+1)$

(B) $\sum_{r=4}^{n} T_r \Rightarrow n+1-4 = (n-3)$

(C) $(a+b+c+d)^{10}$

$$(a+b+c+d)^{10} = \sum 10! \frac{a^p + b^q + c^r + d^s}{p!\,q!\,r!\,s!}$$

$p+q+r+s=10$

$\Rightarrow$ $0+2+4+4 \Rightarrow 4!/2!$

$0+4+2+4$

$0+4+4+2$

$0+0+5+5 = 4!/2!\cdot 2!$

$0+0+0+10 = \frac{4!}{3!}$

$1+1+1+7 = 4!/3!$

$2+2+2+4 = 4!/3!$

$1+3+3+3 = 4!/3!$

distributing 10 chocolate in 4..

0 0 | 0 0 | 0 0 | 0 0 0 0

$$\Rightarrow \frac{13!}{10! \times 3!} = \frac{13 \times \cancel{12}^{2} \times 11}{\cancel{6}} = \boxed{286}$$

1×10^{19}

Date:
Page No.: A4

Que:- $(a+b+c+d)^{10}$ find cofficient of

(A) a^{10} $= 1$

(B) $a^{10}b^2$ $= 0$

(C) $a^3b^3c^3d^3$ $= 0$

(D) $a^2b^2c^2d^4$ $= {}^{10}C_2 \times {}^{8}C_2 \times {}^{6}C_2 \times {}^{4}C_4$

(E) $abcd$ $= 0$

(F) $a^4b^3c^2d$ $= {}^{10}C_4 \times {}^{6}C_3 \times {}^{3}C_2$

(G) $a^p b^q c^r d^s$ where $p+q+r+s \neq 10$ $= 0$

(H) $a^p b^q c^r d^s$ where $p+q+r+s = 10$

$\Downarrow$

$${}^{10}C_p + {}^{10-p}C_q + {}^{10-p-q}C_r$$

Power ka sum hamesha 10 aaega.

${}^{10}C_1 \times {}^{9}C_3 \times {}^{8}C_1 \times 3$

Que:- True or false

$$(a+b+c+d)^{10} = \sum 10! \frac{a^p b^q c^r d^s}{p!\,r!\,q!\,s!}$$

$p, q, r, s \in$ whole no.

$p+q+r+s = 10$

Que:- $p+q+r+s = 20$, $p, q, r, s \in W$ then find

$$\sum \frac{1}{p!\,q!\,r!\,s!}$$

$$(a+b+c+d)^{20} = \sum \frac{20!\, a^p b^q c^r d^s}{p!\,q!\,r!\,s!}$$

$$4^{20} = \sum \frac{20!}{p!\,q!\,r!\,s!}$$

$(-3)^2 \times {}^{10}C_2$

$$\frac{4^{20}}{20!} = \sum \frac{1}{p!\,q!\,r!\,s!}$$

Date:
Page No.: 4

Que:- n like objects, r distinct object 'n' each box can have any object from 0 to n.

$$^{n+r-1}C_{r-1}$$

Que:- find the total number of whole Number Solution:-

(A) $(a+b+c+d) = 100$

$$\frac{103!}{100! \times 3!} = \frac{103 \times 102 \times 101}{6}$$

(B) $a+b+c+d \le 100$

$\hookrightarrow a+b+c+d+e = 100$

$$\frac{104!}{100! \times 4!}$$

(C) $a+b+c-d = 100$

$\hookrightarrow a+b+c \ge 100 \rightarrow \infty$ numbers

(D) $a+b+c+20d = 100$

$a+b+c=100$ $a+b+c=80$, $a+b+c=60$, $a+b+c=40$

$a+b+c=20$

$a+b+c=0$

$$^{102}C_2 + {}^{82}C_2 + {}^{62}C_2 + {}^{42}C_2 + {}^{22}C_2 + 1$$

Que:- $a+b+c+d=100$, $c<40$, $d<40$

C-I $a+b+c+d=100$ $c\geq 40$
$\Rightarrow {}^{63}C_3$

C-II $a+b+c+d=100$ $d\geq 40$
${}^{63}C_3$

C III $c\geq 40 \cap D\geq 40 \Rightarrow {}^{23}C_3$

Total ways $= {}^{103}C_3 - 2\cdot {}^{63}C_3 + {}^{23}C_3$

Que:- $a+b+c+d=100$, $b<20$, $c<20$, $d<20$

C-I $= {}^{83}C_3$ $b\geq 20$
C-II $= {}^{83}C_3$ $c\geq 20$
C-III $= {}^{83}C_3$ $D\geq 20$
C-IV $= {}^{63}C_3$ $b\geq 20$, $c\geq 20$ $A\cap B$
C V $= {}^{63}C_3$ $c\geq 20$, $D\geq 20$ $B\cap C$
C VI $= {}^{63}C_3$ $D\geq 20$, $D\geq 20$ $C\cap D$
C VII ${}^{43}C_3$ $D\geq 20$, $B\geq 20$, $C\geq 20$ $A\cap B\cap B$

Total ways $= 3\cdot {}^{83}C_3 - 3\cdot {}^{63}C_3 + {}^{43}C_3$

ans $=$ Total $- \Sigma AB + A\cap B\cap C$

Page No.: A4

(e) $a+b+c+d = 100$ $a \geq 10,\ b > 11$

Total ways $= {}^{103}C_3$

$a+b+c+d = 78$

$\boxed{{}^{81}C_3}$

(f) $a+b+c+d = 100$ $c < 60$

$a+b+c+d = 59$ $a+b+d = 41$

${}^{62}C_3$ ${}^{43}C_2$

Total ways $= {}^{62}C_3 + {}^{43}C_2$

(g) $a+b+c+d = 100$, $c < 60$, $d < 60$

alternative :-

C-I- $a+b+c'(60)+d = 100$

$a+b+c'+d = 40$ $= {}^{43}C_3$

C-II- $a+b+c'+d+60 = 100$ $= {}^{43}C_3$

C-III $a+b+c+d = 100$ $c \geq 60,\ d \geq 60$

$\Rightarrow 0$

Total ways $= {}^{103}C_3 - 2\,{}^{43}C_3$

Class Mind
Date:
Page No.: A4

Qu:- $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots$
$= \sum a_r x^r$

(A) $a_0 + 8a_3 + 64a_6 + \cdots$
jab 3 ka difference ho to ω ke bare me socho.

$f(2) = a_0 + a_1 2 + a_2 2^2 + a_3 2^3 + \cdots$
$f(2\omega) = a_0 + a_1 2\omega + a_2(2^2\omega^2) + a_3(2^3\omega^3) + \cdots$

$f(2\omega^2) = a_0 + a_1(2\omega) + a_2(2^2\omega^4)$

$\Rightarrow$ $\boxed{\dfrac{f(2) + f(2\omega) + f(2\omega^2)}{3}}$ $\omega = \frac{1}{2} + \frac{i\sqrt{3}}{2}$

$\omega^2 = \frac{1}{2} - \frac{i\sqrt{3}}{2}$

(B) $\cancel{a_0} - \cancel{4a_2 + 16a_4 - 64}a_6 + \cdots$

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots$

$f(2i) = a_0 + 2a_1 i + 4a_2(-1) + \cdots$

real part of $f(2i)$ is ans.

(f) $a_0 + 3a_2 + 5a_4 + \cdots = \frac{A+B}{2}$

$A = a_0 + 2a_1 + 3a_2 + 4a_3 + 5a_4 + \cdots = \sum (r+1) a_r$

$B = a_0 - 2a_1 + 3a_2 - 4a_3 + \cdots = \sum (-1)^r (r+1) a_r$

$f(x) = \sum a_r \cdot x^r$

$f'(x) = \sum r \cdot a_r x^{(r-1)}$

$= x f'(x) = \sum r \cdot a_r \cdot x^r$

$G(x) = f(x) + f'(x) = \sum (r+1) a_r x^r$

Ans $= \frac{f(1) + 1 f'(1) + (f(-1) - 1 (f'(-1))}{2}$

(g) $a_0 - 3a_2 + 5a_4 - 7a_6 + \cdots$

$\Rightarrow$ Real part $[f(i) + i f'(i)]$

(h) $a_0 + 2a_1 7 + 3a_2 7^2 + 4a_3 7^3 + \cdots$

$\sum (r+1) a_r 7^r = \sum r a_r 7^r + \underbrace{\sum a_r 7^r}_{f(7)}$

$f(x) = \sum a_r x^r$

$\frac{f'(x)}{x} = \sum r a_r x^r$

$7 f'(7) = \sum r a_r 7^r$

Ans $= 7 f'(7) + f(7)$.

Date
Page No.

(c) $a_1 + 2a_2(5) + 3a_3(5^2) + \dots$

$= \sum r(a_r) \cdot 5^{(r-1)}$

$f(x) = \sum a_r x^r$

differentiating both side by x.

$f'(x) = \sum (a_r) \cdot r \cdot x^{(r-1)}$

$\boxed{ans = f'(5)}$

(d) $a_1 + 2a_2 + 3a_3 + \dots$

$= \sum (a_r)\, r$

$f(x) = \sum (a_r)\, x^r$

$f'(x) = \sum r \cdot a_r\, x^{r-1}$

$\underline{f'(1)} = \sum (a_r) \cdot r$

(e) $a_0 + 2a_1 + 3a_2 + 4a_3 + \dots$

$\sum (r+1)\, a_r$

$= \sum r \cdot a_r \quad \sum a_r$

$= \underline{f(1)} + f(1)$

$f(x) = \sum (a_r) x^r$

$f'(x)$

Date:
Page No.:

$$= \sum a_r \frac{x^{r+1}}{r+1} \Big|_0^x$$

$$H(x) \int_0^x f(x)\,dx = \sum a_0 \frac{x^{r+1}}{r+1}$$

$$H(3) \int_0^x f(3)\,dr \quad \sum \frac{1}{(r+1)} a_0 \; 3^{(r+1)}$$

① $$\frac{a_0}{1\cdot 2} + \frac{a_1}{2\cdot 3} + \frac{a_2}{3\cdot 4} + \cdots$$

$$= \sum \frac{a_r}{(r+1)(r+2)}$$

$$f(x) = \sum a_r x^r$$

$$\int_0^x f(x) = \int \sum a_r x^r$$

$$H(x) = \int_0^x f(x)\,dx = \sum a_r \frac{x^{r+1}}{r+1}$$

$$G(x) = \int_0^x H(x)\,dx = \sum \frac{a_r}{(r+1)(r+2)} x^{r+2}$$

$$G(1) = \int_0^1 H(x)\,dx = \int_0^x \left(\int_0^x f(x)\,dx\right) = \sum \frac{a_r}{(r+1)(r+2)}$$

(I) $2\cdot1\cdot a_2 + 3\cdot2\cdot a_3 + 4\cdot3\cdot a_4 + \cdots$

Sol

$f'(x) = 1a_1 + 2\cdot a_2 x + 3\cdot a_3 x^2 + \cdots$

$\boxed{f''(x) = 0 + 2\cdot1\,a_2 + 3\cdot2\cdot a_3 x + 4\cdot3\,a_4}$

$\boxed{f''(1)}$ ans.

(J) $a_1 + 4a_2 + 9a_3 + 16a_4 + \cdots$

~~$f(x) = a_1 + 2a_2 x + 3\cdot a_3 x^2 + 4 a_4 x^3$~~

$\Rightarrow \sum r^2 (a_r)$

$f(x) = \sum a_r x^r$

$f'(x) \sum r (a_r) x^{r-1}$

$x f'(x) \sum r \cdot a_r x^r$

$f'(x) + x f''(x) \sum r^2 a_r x^{r-1}$

$f'(1) + 1 f''(1) \sum r^2 a_r$

(K) $\frac{a_0}{1}(3) + \frac{a_1}{2}(3)^2 + \frac{a_2}{3}(3)^3 + \cdots$

$= \sum \frac{1}{(r+1)} a_r \cdot 3^{r+1}$

$f(x) = \sum a_r x^r$

$\int_0^x f(x)\,dx = \int_0^x \sum a_r x^r$

Date:
Page No.: A4

Que:- $f(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + \cdots$

find :- $\frac{A_0}{1\cdot 2} + \frac{A_1}{6\cdot 7} + \frac{A_2}{10\cdot 11} + \frac{A_3}{16\cdot 17} + \cdots$

$$\sum \frac{a_r}{(5r+1)(5r+2)}$$

$$f(x) = \sum a_r x^r$$

$$f(x^5) = \sum a_r x^{5r}$$

$$H(x) = \int_0^x f(x^5) = \int \sum \frac{a_r x^{(5r+1)}}{(5r+1)}$$

$$G(x) = \int_0^x \left(\int_0^x f(x^5) \right) = \sum \frac{a_r \, x^{(5r+2)}}{(5r+1)(5r+2)}$$

$$G(1) \quad \int_0^1 \left(\int_0^1 f(x^5) \right) = \sum \frac{a_r}{(5r+1)(5r+2)}$$

Que:- $f(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + \cdots$

find :- $\frac{A_0}{1\cdot 2\cdot 3} + \frac{A_1}{2\cdot 3\cdot 4} + \frac{A_2}{3\cdot 4\cdot 5} + \frac{A_3}{4\cdot 5\cdot 6} + \frac{A_4}{5\cdot 6\cdot 7} + \cdots$

$$\sum \frac{a_r}{(r+1)(r+2)(r+3)}$$

$$= \int_0^1 \int_0^x \int_0^x f(x)$$

Date:
Page No. 44

Que:- $f(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \dots$

find :- $\frac{A_0}{1\cdot 2\cdot 5} + \frac{A_1}{2\cdot 3\cdot 7} + \frac{A_2}{3\cdot 4\cdot 9} + \dots$

$$\sum \frac{A_r}{(r+1)(r+2)(5+2r)}$$

$f(x) = \sum A_r x^r$

$$\int_0^x f(x) = \sum \frac{A_r}{(r+1)} x^{r+1}$$

$$H(x) = \int_0^x \int_0^x f(x)\,dx\,dx = \sum \frac{A_r}{(r+1)(r+2)} x^{r+2}$$

$$H(x^2) = \sum \frac{A_r}{(r+1)(r+2)} x^{2r+4}$$

$$I(x) = \int_0^x H(x^2) = \sum \frac{A_r}{(r+1)(r+2)(2r+5)} x^{2r+5}$$

$$I(1) = \sum \frac{A_r}{(r+1)(r+2)(2r+5)} = \int_0^1 H(x^2) = \int_0^{x^2}\left[\int f(x)\,dx\right]dx$$

(m) $\frac{a_0}{1} + \frac{a_1}{4} + \frac{a_2}{7} + \frac{a_3}{10} + \dots$

$\Rightarrow \sum \frac{a_r}{(3r+1)}$

$f(x) = \sum a_r x^r$

$f(x^3) = \sum a_r x^{3r}$

$H(x) \int_0^x f(x^3)\,dx = \sum \frac{a_r}{(3r+1)} x^{(3r+1)}$

$H(1) = \int_0^1 f(x^3)\,dx = \sum \frac{a_r}{(3r+1)}$

(n) $1 \cdot 2 \cdot a_0 + 2 \cdot 3 \cdot a_1 + 3 \cdot 4 \cdot a_2 + \dots$

$\sum (r+1)(r+2)\, a_r$

$f(x) = \sum a_r x^r$

$x^2 f(x) = \sum a_r x^{r+2}$

$x^2 f'(x) = \sum (r+2)\, a_r x^{(r+1)}$

$H(x) = x^2 f'(x) + 2x f'(x) = \sum (r+1)(r+2)\, a_r x^r$

$H(1) = 1\, f(x)$

Class Mind
Date:
Page No.: A4

Qu:- find :- $\frac{A_0}{3} - \frac{A_1}{4} + \frac{A_2}{5} - \frac{A_3}{6} + \frac{A_4}{7} - - -$

$$\sum \frac{A_r (-1)^r}{(r+3)}$$

$$f(x) = \sum A_r x^r$$

$$x^2 \cdot f(x) = \sum A_r x^{r+2}$$

$$G(x) \int_0^x x^2 f(x) = \int \sum \frac{A_r x^{r+3}}{r+3}$$

$$\Rightarrow \cancel{\int_0^1 f(x)} \quad \cancel{\sum \frac{A_r}{(r+3)}}$$

$$\Rightarrow G(-1) = \int_0^{-1} x^2 f(x) = -\sum \frac{A_r (-1) \cdot (-1)^3}{r+3}$$

$$= G(-1) = -\int_0^{-1} x^2 f(x) = \sum \frac{A_r (-1)}{r+3} = \int_{-1}^{0} x^2 f(x)\, dx$$

Qu:- find :- $\frac{A_0}{3} + \frac{A_2}{5} + \frac{A_4}{7} + \frac{A_6}{9} + - - - = \frac{P+q}{2}$

this come from :- $\frac{A_0}{3} + \frac{A_1}{4} + \frac{A_2}{5} + \frac{A_3}{6} + \frac{A_4}{7} + - - - = P$

$$\frac{A_0}{3} - \frac{A_1}{4} + \frac{A_2}{5} - \frac{A_3}{6} + - - - = Q$$

$$P = \sum \frac{A_r}{r+3} = \int_0^1 x^2 f(x)\, dx$$

$$q = \sum (-1)^r \frac{A_r}{r+3} = \int_{-1}^{0} x^2 f(x)\, dx$$

Que:- $\sum r(r-1)\,{}^nC_r$

$(1+x)^n = \sum {}^nC_r\, x^r$

$n(1+x)^{n-1} = \sum {}^nC_r \cdot r \cdot x^{r-1}$

$n\cdot(n-1)(1+x)^{n-2} = \sum r\cdot(r-1)\,{}^nC_r\, x^{r-2}$

$n\cdot(n-1)\,2^{(n-2)} = \sum r\cdot(r-1)\,{}^nC_r$

Que:- $\sum_{r=0}^{n} r^2\,{}^nC_r$

$(1+x)^n = \sum {}^nC_r\, x^r$

$n(1+x)^{n-1} = \sum {}^nC_r \cdot r \cdot x^{r-1}$

$x\cdot n(1+x)^{n-1} = \sum {}^nC_r \cdot r \cdot x^r$

$n\cdot(1+x)^{n-1} = \sum r^2\cdot {}^nC_r\, x^{r-1}$
$+ nx(n-1)(1+x)^{n-2}$

$\Rightarrow n\,2^{(n-1)} + n(n-1)\,2^{n-2} = \sum r^2 \cdot {}^nC_r$

Que:- $\sum_{r=0}^{n} \frac{1}{(r+1)} {}^nC_r$

$$(1+x)^n = \sum {}^nC_r x^r$$

$$\int_0^x (1+x)^n dx = \sum \frac{{}^nC_r}{(r+1)} x^{r+1}$$

$$\int_0^1 (1+x)^n dx = \sum \frac{1}{(r+1)} {}^nC_r$$

$$= \frac{(1+x)^{n+1}}{n+1}\Big|_0^1 = \frac{2^{n+1}}{n+1} = \sum \frac{1}{(r+1)} {}^nC_r$$

Que:- $\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)} {}^nC_r$

$$(1+x)^n = \sum {}^nC_r x^r$$

$$\int_0^x (1+x)^n dx = \sum \frac{1}{(r+1)} {}^nC_r x^{r+1}$$

$$\int_0^x \left(\int_0^x (1+x)^n dx \right) dx = \sum \frac{1}{(r+1)(r+2)} {}^nC_r x^{r+2}$$

$$\Rightarrow \int_0^x \left(\frac{(1+x)^{n+1}}{n+1} \right)\Big|_0^1$$

$$= \int_0^x \int \frac{(1+x)^{n+1}}{n+1} \Rightarrow \frac{1}{n+1} \left(\frac{(1+x)^{n+2}}{n+2} - \int_0^x \frac{x}{n+1} \Big|_0^1 \right)$$

$$= \frac{2^{n+2}}{(n+1)(n+2)} - \frac{1}{(n+1)(n+2)} + \frac{1}{n+1}$$

Class Mind
Date:
Page No.: A4

Ans:- $\dfrac{\int_0^1 x^2 f(x)\,dx + \int_{-1}^0 x^2 f(x)\,dx}{2}$

Que:- find :- $\dfrac{A_0}{3} - \dfrac{A_2}{5} + \dfrac{A_4}{7} - \dfrac{A_6}{9} + \cdots$

Que:- find :- $1A_0 + 4A_1 + 7A_2 + 10A_3 + \cdots$

$= \sum A_r (3r+1)$

$f(x) = \sum A_r x^r$

$\Rightarrow f(x^3) = \sum A_r x^{3r}$

$\Rightarrow x^2 f(x^3) = \sum A_r x^{3r+2}$

$\Rightarrow x\, f'(x^3) = \sum (3r+1) A_r x^{3r}$

$\Rightarrow f(1) + f'(1) = \sum (3r+1) A_r$

Qu: $(1-x)^{10}$

$$\left|\frac{T_{r+1}}{T_r}\right| \geq 1$$

$$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} \geq 1 \Rightarrow \frac{n-r+1}{r} \geq 1$$

$$\Rightarrow \frac{10-r+1}{r} \geq 1$$

$$\Rightarrow \frac{11-r}{r} \geq 1$$

$$2r \leq 11$$

$$r \leq 5.5$$

$$\boxed{r=5}$$

T_6 greatest cofficient

(C) $(1+x)^{11}$

$$\Rightarrow \frac{11-r+1}{r} \geq 1$$

$$\Rightarrow \frac{12-r}{r} \geq 1$$

$$\Rightarrow 2r \leq 12$$

$$\boxed{r \leq 6}$$

${}^{11}C_6$ = T_7 would be greatest

(D) $(1-x)^{11}$

$$\frac{11-r+1}{r} \geq 1$$

$$r \leq 6$$

$$\boxed{r=6} = {}^{11}C_6$$

Page No.: 44

(E) $(2+3x)^{10} \Rightarrow 2^{10}\left(1+\frac{3x}{2}\right)^{10}$

$\frac{T_{r+1}}{T_r} \geq 1$

$\frac{{}^{10}C_r \left(\frac{3}{2}\right)^r}{{}^{10}C_{r-1} \left(\frac{3}{2}\right)^{r-1}} \geq 1$

$\frac{10-r+1}{r} \times \frac{3}{2} - 1 \geq 0$

$\frac{33-5r}{2r} \geq 0$

$33 \geq 5r$

$r \leq 6.6$

$\boxed{r=6}$

greatest cofficient $= {}^{10}C_6 \times 2^{10} \times \left(\frac{3}{2}\right)^6$

(F) $(2-3x)^{10} \Rightarrow$ same answer as previous.

Qu:- Greatest term according To magnitude only:-

(A) $(1+x)^{10}$; if $x = 1/4$

$\frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{10-r+1}{r} \geq 1$

$2r \leq 11$

$r \leq 5.5$

$\boxed{r=5}$

greatest term:- ${}^{10}C_5 \cdot 1^5 \cdot x^5$

$= {}^{10}C_5 \cdot \left(\frac{1}{4}\right)^5$