Binomial theorem

* Any algebric expression which contains two dissimal terms is called binomial expression

$(a+b)^0 = 1$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ → degree = 3
$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

* sabhi terms ki degree 4 hai

Pascal's triangle (for cofficient)

1	← $(a+b)^0$
1 1	← $(a+b)^1$
1 2 1	← $(a+b)^2$
1 3 3 1	← $(a+b)^3$
1 4 6 4 1	← $(a+b)^4$
1 5 10 10 5 1	← $(a+b)^5$
1 6 15 20 15 6 1	← $(a+b)^6$
1 7 21 35 35 21 7 1	← $(a+b)^7$
1 8 28 56 70 56 28 8 1	← $(a+b)^8$

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~~Application of binomial theorem~~

Sum of binomial cofficients

☆ $\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots {}^nC_n = 2^n}$

Proof:-

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y + \dots + {}^nC_n \cdot x^0 y^n$$

if $x, y = 1$

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$$

☆ Simillarly,

$${}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + \dots {}^{n+1}C_{n+1} = 2^{n+1}$$

☆ ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 + \dots + {}^nC_n = 0$

$\Rightarrow {}^nC_0 + {}^nC_2 + {}^nC_4 + {}^nC_6 + \dots = {}^nC_1 + {}^nC_3 + {}^nC_5 \dots$

☆ $\boxed{2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + {}^nC_6 \dots}$

$\Rightarrow$ if $(x^2+x+1)^n = a_0 + a_1 x + a_2 x^2 + a_3 a x^3 + \dots$

then find the sum of $= a_0 + a_1 + a_2 + a_3 + \dots a_n$

putting $x = 1$

Sum of cofficients $= 3^n$

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Finding remainder using binomial theorem

Que what is the remainder when 5^{99} divided by 13.

Ignore this

$5^{99} = 5(\cancel{2}6-1)^{49} \equiv (-5)^{9}$

$\Rightarrow \quad 5({}^{49}C_0\, 26^{49} - {}^{49}C_1\, 26^{48} + {}^{49}C_2\, 26^{47} + \cdots$
$\cdots - {}^{49}C_{49})$

$\Rightarrow$ all the terms are devisble by 13 except $= -5 = 8$ remainder

Que:- Remainder = ? when $7^{103} \div 25$

$7^{103} = 7(50-1)^{51}$
$\equiv -7$
remainder $= 18$

Integral and fractional part

Que:- if $(2+\sqrt{3})^n = I + f$ where I and n are +ve integer and $0 < f < 1$ then $(1-f)(I+f) = ?$

$I + f = (2+\sqrt{3})^n$ — (i)

$f' = (2-\sqrt{3})^n$ — (ii)

adding both
$I + f + f' =$ Integer
hence, $f + f' = 1$
$f' = \underline{\underline{1-f}}$

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cofficient of x^7 in $(1+x)^{20}$

7 terms se x lena hai to $^{20}C_7$

$\Rightarrow$ cofficient of $x^0 = {}^{20}C_0$

$\Rightarrow$ cofficient of $x^3 = {}^{20}C_3$

$$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + {}^{20}C_3 x^3 \ldots + {}^{20}C_{20} x^{20}$$

$\boxed{\text{Taylor Series}}$

Expansion of $f(x) = e^x$

$f(x) = e^x = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \ldots$

$F(0) = e^0 = 1 = A_0$

differentiate kar diya uppar ki equ^n ko

$F'(0) = e^x = \frac{F'(0)}{1!} = 1 = A_1$

$f''(0) = e^x = \frac{f''(0)}{2!} = \frac{1}{2!}$

By Mc Lorin series

$F'''(0) = e^x = \frac{F'''(0)}{3!} = \frac{1}{3!}$

expansion :- $1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \ldots$

cofficients

1

1 1

1 2 1 $\Rightarrow (a+b)^2 \Rightarrow$ ${}^2C_0 + {}^2C_1 + {}^2C_2$

1 3 3 1 $(a+b)^3 =$ ${}^3C_0$ ${}^3C_1$ ${}^3C_2$ ${}^3C_3$

1 4 6 4 1 $(a+b)^4 =$ ${}^4C_0$ ${}^4C_1$ ${}^4C_2$ ${}^4C_3$ ${}^4C_4$

Binomial expensions

$$\boxed{(a+b)^n = {}^nC_0 \cdot a^n \cdot b^0 + {}^nC_1 \cdot a^{n-1} \cdot b + {}^nC_2 \cdot a^{n-2} \cdot b^2 + {}^nC_3 \cdot a^{n-3} \cdot b^3 + \cdots + {}^nC_n \cdot a^0 \cdot b^n}$$

$$(a-b)^n = [a+(-b)]^n = {}^nC_0 \cdot a^n \cdot (-b)^0 + {}^nC_1 \cdot a^{n-1} \cdot (-b)^1 \ldots$$

$-------$

~~Ques:-~~ ~~$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$ gives the value =~~

~~$(\sqrt{2})^6 + 6(\sqrt{2})^5 +$ ..~~

General term

$$(x+y)^n = {}^nC_0\, x^n y^0 + {}^nC_1\, x^{n-1} y^1 + {}^nC_2\, x^{n-2} y^2 \ldots$$

$1 \to r$

$\boxed{{}^nC_r\, x^{n-r} y^r}$ → This term is valid for all the terms

$$\boxed{T_{r+1} = {}^nC_r\, x^{n-r} y^r}$$

Now,

$$(1-f)(I+f)$$
$$= (2+\sqrt{3})^n (2-\sqrt{3})^n$$
$$= (1)^n = 1$$

Identifying larger or smaller part

Que:- Larger of 99^{50} and 100^{50} and 101^{50}

$$99^{50} = (100-1)^{50}$$
$$= {}^{50}C_0\, 100^{50} (-1)^0 + {}^{50}C_1\, 100^{49} (-1)^{49} + {}^{50}C_2\, 100^{48} (-1)^{48} + \cdots$$

$$(101)^{50} = (100+1)^{50}$$
$$= {}^{50}C_0\, 100^{50}\, 1^0 + {}^{50}C_1\, 100^{49}\, 1 + {}^{50}C_2 \cdot 100^{48} \cdot 1^2 + \cdots$$

$$(101)^{50} - (99)^{50} = 2\left[{}^{50}C_1\, 100^{49} + {}^{50}C_3\, 100^{47} + \cdots\right.$$

$$= 2 \times 50 \times 100^{49} + \text{something}$$
$$= 100^{50} + \text{something (+ve)}$$

Hence $(101)^{50} > (99)^{50} + (100)^{50}$

$f(x) = \sin x \quad = 0$

$f'(x) = \cos x \quad = \frac{1}{1!}$

$f''(x) = -\sin x \quad = \frac{0}{2!}$

$f'''(x) = -\cos x \quad = \frac{-1}{3!}$

$f''''(x) = \sin x \quad = \frac{0}{4!}$

$$\boxed{\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}}$$

$f(x) = \cos x \quad = 1$

$f'(x) = -\sin x \quad = 0$

$f''(x) = -\cos x \quad = -1/2!$

$f'''(x) = \sin x \quad = 0/3!$

$f''''(x) = \cos x \quad = 1/4!$

$$\boxed{\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} - - -}$$

$f(x) = \ln(1+x)$

$f(x) = e^{ix}$

$f(x) = \tan x$

$f(x) = (1+x)^{-1}$

$f(x) = a^x$

Middle Term

1. If n even hai ~~to~~, then their is only one middle term.

$(x+y)^n \to (n+1)$ terms

$$\boxed{\text{middle term for } n = \left(\frac{n+2}{2}\right)^{th}}$$

2. If n is odd then there are two middle terms.

$$\boxed{\text{middle term} = \left(\frac{n+1}{2}\right)^{th} \text{ and } \left(\frac{n+1}{2}+1\right)^{th}}$$

Numerically Greatest Term (cofficient)

sign ko weitage nahi dena hai

let Term_{r+1} is greatest then it will be greater then left and right term.

$$|T_r| \le |T_{r+1}| \ge |T_{r+2}|$$

$$\left|{}^nC_{r-1} \cdot x^{n-r+1} \cdot y^{r-1}\right| \le \left|{}^nC_r \cdot x^{n-r} \cdot y^r\right|$$

$$= \left|\frac{n!}{(r-1)!\,(n-r+1)!} \cdot x\right| \le \left|\frac{n!}{r!\,(n-r)!} \cdot y\right|$$

$$= \left|\frac{x}{n-r+1}\right| \le \frac{|y|}{|r!|}$$

$\boxed{\text{expand } e^{ix}}$

$$\cancel{f(x) = e^{ix} \Rightarrow f(0) =}$$
$$\cancel{f'x = e^{ix}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{ix} = 1 + ix - \frac{x^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \cdots$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right) \left(ix - \frac{ix^3}{3!} + \frac{ix^5}{5!} + \cdots\right)$$

$$\boxed{e^{ix} = \cos x + i\sin x}$$

Expand $\tan x$

$f(x) = \tan x \quad f(0) = 0$

$f'(x) = \sec^2 x = 1 + \tan^2 x = f(0) = 1$

$f''x = 2\tan x \sec^2 x = 2\tan x + 2\tan^3 x = 0$

$f'''x = 2 \sec^2 x + 6\tan^2 x \sec^2 x$
$= 2 + 2\tan^2 x + 6\tan^2 x + 6\tan^4 x =$

$f''''x = 0 + 16\sec^2 x + 24\tan^3 x (\sec^2 x)$
$= 16 + 16\tan^3 x + 24\tan^3 x + 24\tan^5 x \Rightarrow f(0) = 0$

$A_0 = 0, \; A_1 = 1, \; A_2 = 0, \; A_3 = \frac{2}{3!}, \; A_5 = \frac{2}{15}$

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Que:- $\sum_{r=0}^{10} (-1)^r \, {}^{10}C_r$

$\Rightarrow {}^{10}C_0 - {}^{10}C_1 + {}^{10}C_2 - {}^{10}C_3 + \cdots \; {}^{10}C_{10}$

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + {}^{10}C_3 x^3 + \cdots$

$f(-1) = {}^{10}C_0 - {}^{10}C_1 + {}^{10}C_2 - {}^{10}C_3 + \cdots$

Ans = 0

Que:- $\sum_{r=0}^{10} {}^{10}C_{2r}$

${}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + {}^{10}C_6 + {}^{10}C_8 + {}^{10}C_{10}$

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 + \cdots$

$\frac{f(1) - f(-1)}{2} = \frac{2^{10}}{2} = \boxed{2^9}$

Que:- $\sum_{r=0}^{10} r \cdot {}^{10}C_r = \sum_{r=0}^{10} 10 \, {}^{9}C_{r-1}$

$10\left({}^9C_{-1} + {}^9C_0 + {}^9C_1 + {}^9C_2 + \cdots \; {}^9C_9\right)$

$= \boxed{10 \cdot 2^9}$

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Que:- $\sum_{r=0}^{20} {}^{25}C_r \, {}^{20}C_r$

$\Rightarrow {}^{25}C_0 \, {}^{20}C_0 + {}^{25}C_1 \, {}^{20}C_1 + {}^{25}C_2 \, {}^{20}C_2 + \cdots + {}^{25}C_{20} \, {}^{20}C_{20}$

$\Rightarrow {}^{25}C_0 \, {}^{20}C_{20} + {}^{25}C_1 \, {}^{20}C_{19} + {}^{25}C_2 \, {}^{20}C_{18} + \cdots$

$\Rightarrow \boxed{{}^{45}C_{20}}$

Que:- $\sum_{r=0}^{20} \left({}^{20}C_r\right)^2$

$- {}^{10}C_{10} \, {}^{20}C_0$

${}^{20}C_0 \, {}^{20}C_0 + {}^{20}C_1 \, {}^{20}C_1 + \cdots + {}^{20}C_{20} \, {}^{20}C_{20}$

$= \boxed{{}^{40}C_{20}}$

apple

Que:- $\sum_{r=1}^{20} {}^{30}C_r \, {}^{10}C_r$

${}^{30}C_1 \, {}^{10}C_1 + {}^{30}C_2 \, {}^{10}C_2 + \cdots + {}^{30}C_{20} \, {}^{10}C_{20}$

$= {}^{30}C_1 \, {}^{10}C_9 + {}^{30}C_2 \, {}^{10}C_8 + {}^{30}C_3 \, {}^{10}C_7 +$

$\boxed{{}^{40}C_{10} - 1} \rightarrow ?$

$- {}^{30}C_{20} \, {}^{20}C_0$

Que:- $\sum_{r=0}^{10} {}^{15+r}C_{10+r}$

${}^{15}C_{10} + {}^{16}C_{11} + {}^{17}C_{12} + {}^{18}C_{13} + \cdots + {}^{25}C_{20}$

$\Rightarrow {}^{5}C_0 + {}^{6}C_1 + \cdots + {}^{14}C_9 + \left({}^{15}C_{10} + \cdots + {}^{25}C_{20}\right)$

$\Rightarrow \boxed{{}^{26}C_{20} - {}^{15}C_9}$

$$\boxed{r\,{}^{n}C_r \Rightarrow n\,{}^{n-1}C_{r-1}}$$

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Qu:- $\sum_{r=0}^{10} r^2 \cdot {}^{10}C_r = \sum r \cdot (r \cdot {}^{10}C_r)$

$= 10\sum r \cdot {}^{9}C_{r-1}$

$= 10\sum (r-1)\, {}^{9}C_{r-1} + {}^{9}C_{r-1}$

$= 10\sum 9 \cdot {}^{8}C_{r-2} + 10\sum {}^{9}C_{r-1}$

$= 90\sum {}^{8}C_{r-2} + 10\sum {}^{9}C_{r-1}$

$= 90\left[{}^{8}C_0 + {}^{8}C_1 + \cdots + {}^{8}C_8\right] + 10\left[{}^{9}C_0 + {}^{9}C_1 + \cdots {}^{9}C_9\right]$

$= 90 \times 2^8 + 10 \times 2^9$

Qu:- $\sum_{r=0}^{10} (r^2 + r + 1)\, {}^{12}C_r$

$\sum_{r=0}^{10} r^2\, {}^{12}C_r + \sum_{r=0}^{10} r\, {}^{12}C_r + \sum_{r=0}^{10} {}^{12}C_r$

$C = {}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + \cdots {}^{12}C_{10}$

$= \boxed{2^{12} - {}^{12}C_{11} - 1}$

$B = 12\left[{}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \cdots + {}^{11}C_9\right]$

$= 12\left[2^{11} - 12\right]$

$A = \sum r \cdot (r\, {}^{12}C_r) - \sum r \cdot (12\, {}^{11}C_{r-1})$

$= 12\sum r \cdot {}^{11}C_{r-1}$

$= 12\sum (r-1)\, {}^{11}C_{r-1} + 12\sum {}^{11}C_{r-1}$

$= 11 \times 12 \sum_{0}^{10} {}^{10}C_{r-2} + 12\sum_{r=0}^{10} {}^{11}C_{r-1}$

$= 11 \times 12\left[2^{10} - 11\right] + 12\left[2^{11} - 12\right]$

$$\boxed{r\,{}^nC_r \Rightarrow n\,{}^{n-1}C_{r-1}}$$

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Que:- $\sum_{r=0}^{10} r^2 \cdot {}^{10}C_r = \sum r \cdot (r \cdot {}^{10}C_r)$

$= 10 \sum r \cdot {}^{9}C_{r-1}$

$= 10 \sum (r-1)\, {}^{9}C_{r-1} + {}^{9}C_{r-1}$

$= 10 \sum 9 \cdot {}^{8}C_{r-2} + 10 \sum {}^{9}C_{r-1}$

$= 90 \sum {}^{8}C_{r-2} + 10 \sum {}^{9}C_{r-1}$

$= 90 [{}^{8}C_0 + {}^{8}C_1 + \cdots + {}^{8}C_8] + 10 [{}^{9}C_0 + {}^{9}C_1 + \cdots {}^{9}C_9]$

$= 90 \times 2^8 + 10 \times 2^9$

Que:- $\sum_{r=0}^{10} (r^2 + r + 1)\, {}^{12}C_r$

$\sum_{r=0}^{10} r^2\, {}^{12}C_r + \sum_{r=0}^{10} r\, {}^{12}C_r + \sum_{r=0}^{10} {}^{12}C_r$

$C = {}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + \cdots {}^{12}C_{10}$

$= \boxed{2^{12} - {}^{12}C_{11} - 1}$

$B = 12 [{}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \cdots + {}^{11}C_9]$

$= 12 [2^{11} - 12]$

$A = \sum r \cdot (r\, {}^{12}C_r) - \sum r \cdot (12\, {}^{11}C_{r-1})$

$= 12 \sum r \cdot {}^{11}C_{r-1}$

$= 12 \sum (r-1)\, {}^{11}C_{r-1} + 12 \sum {}^{11}C_{r-1}$

$= 11 \times 12 \sum_{0}^{10} {}^{10}C_{r-2} + 12 \sum_{r=0}^{10} {}^{11}C_{r-1}$

$= 11 \times 12 [2^{10} - 11] + 12 [2^{11} - 12]$

Ans = $A + B + C$

~~$162(2^{10}-1) + 12[$~~

Que:- $\sum_{r=0}^{10} (r^2+r+1)\ {}^{10}C_r$

$\sum_{r=0}^{10} r^2\ {}^{10}C_r$

$= \sum_{0}^{10} r\cdot 10\ {}^{9}C_{r-1}$

$= 10\sum (r-1)\ {}^{9}C_{r-1} + 10\sum {}^{9}C_{r-1}$

$= 9\times 10 \sum_{r=0}^{10} {}^{8}C_{r-2}$

$= 90\times 2^{8} + 10\times 2^{9}$

$= 2^{8}\times 10\,[9+2]$

$= 11\times 10\times 2^{8} = A$

$\sum r\ {}^{10}C_r$

$= 10\times 2^{9} = B$

$\sum_{r=0}^{10} 1\cdot {}^{10}C_r$

$= 2^{10} = C$

$A+B+C =$ Ans

Qu:- $\sum_{r=0}^{n} {}^{n}C_{r} = \boxed{2^{n}}$

☆ $\sum_{r=0}^{n} r\,{}^{n}C_{r} = \boxed{n\cdot 2^{n-1}}$

☆ $\sum_{r=0}^{n} r^{2}\,{}^{n}C_{r} = n(n-1)\cdot 2^{(n-2)} + n\cdot 2^{(n-1)}$

$\Downarrow$

$\boxed{n(n+1)\,2^{n-2}}$

☆ $\sum_{r=0}^{n} r^{3}\,{}^{n}C_{r} \Rightarrow n^{2}(n+3)\,2^{n-3}$

☆ $\sum_{r=0}^{n} (-1)^{r}\,{}^{n}C_{r} \Rightarrow 0$

☆ $\sum (-1)^{r}\, r\,{}^{n}C_{r}$

$\sum (-1)^{r}\, n\,{}^{n-1}C_{r-1}$

$\Rightarrow n\sum_{r=0}^{n} (-1)^{r}\,{}^{n-1}C_{r-1}$

$= n\left[-{}^{n-1}C_{0} + {}^{n-1}C_{1} - {}^{n-1}C_{2} + \cdots\right]$

$= -n\left[{}^{n-1}C_{0} - {}^{n-1}C_{1} + {}^{n-1}C_{2} - \cdots\right]$

$(1+x)^{n}$ $\in x=-1$

$= -n\times 0 = \boxed{0}$

Que:- True or false

$1 + \omega^2 = -\text{cis}\,60^\circ$

$1 + \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 \Rightarrow 1 + \left(\frac{1}{4} - \frac{3}{4} - \frac{i\sqrt{3}}{2}\right)$

$= \frac{5}{4} - \frac{3}{4} + \frac{i\sqrt{3}}{2}$

$= \frac{1}{2} + \frac{i\sqrt{3}}{2}$

$= \cos 60^\circ + i \sin 60^\circ$

Que:- Evaluate

$$\sum_{r=0}^{n} {}^nC_r (2)^r$$

$\Rightarrow {}^nC_0 \cdot + {}^nC_1 2 + 4 \cdot {}^nC_2 + 8 \cdot {}^nC_3 + \cdots {}^nC_n 2^n$

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \cdots$

$f(2) = \text{ans}$

$= \boxed{3^n}$ ans.

Que:- $\sum r \, {}^nC_r (2)^r$

$n \sum {}^{(n-1)}C_{(r-1)} 2^{(r)}$

$n [\cancel{{}^{n-1}C_{r-1}} + \cancel{2^{n-1}} \; 2 \cdot {}^{n-1}C_0 + 2^2 \, {}^{n-1}C_1 + \cdots {}^{n-1}C_{n-1} 2^n$

$\boxed{2n\,[3^{n-1}]}$

$$\sum_{r=0}^{n} (-1)^r r^2 \, {}^nC_r = 0$$

$$\sum_{r=0}^{n} (-1)^r r^3 \, {}^nC_r = 0$$

$$\sum_{r=0}^{n} (3r^2 + r + 1) \, {}^nC_r$$

$$= \sum_{r=0}^{n} 3r^2 \, {}^nC_r + \sum_{r=0}^{n} r \, {}^nC_r + \sum_{r=0}^{n} {}^nC_r$$

$$= 3 \cdot n(n+1) \cdot 2^{n-2} + n \cdot 2^{n-1} + 2^n$$

$$= 2^{n-2} [3n^2 + 3 + 2n + 4]$$

$$= 2^{n-2} [3n^2 + 2n + 7]$$

Que:- True or false

$$1 + \omega = \text{CIS } 60^\circ$$

$$\Rightarrow \omega = -\frac{1}{2} + i\sqrt{3}$$

$$1 + \left(\frac{-1}{2} + \frac{i\sqrt{3}}{2} \right)$$

$$= \frac{1}{2} + \frac{i\sqrt{3}}{2} = \cos 60^\circ + i \sin 60^\circ$$

Hence True.

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Que:- $\sum_{r=0}^{n} ({}^nC_r)({}^nC_r)$

${}^nC_0\,{}^nC_0 + {}^nC_1\,{}^nC_1 + {}^nC_2\,{}^nC_2 + \cdots$

${}^nC_0\,{}^nC_n + {}^nC_1\,{}^nC_{n-1} + {}^nC_2\,{}^nC_{n-2} + \cdots$

$\Rightarrow \boxed{{}^{2n}C_n}$

Que:- $\sum_{r=0}^{n} r({}^nC_r)^2$

$= \sum_{r=0}^{n} n\,{}^{n-1}C_{r-1}\,{}^nC_r$

$= n\sum {}^{n-1}C_{r-1}\,{}^nC_r$

$n\left[{}^{n-1}C_0\,{}^nC_1 + {}^{n-1}C_1\,{}^nC_2 + \cdots\right]$

$n\left[{}^{2n-1}C_{n-1}\right]$

Que:- $\sum r^2({}^nC_r)^2$

$\sum (r\,{}^nC_r)^2$

$= n^2\sum ({}^{n-1}C_{r-1})^2$

$= n^2\left[{}^{2n-2}C_{n-1}\right]$

☆ $\sum_{r=0}^{n} T_r = T_0 + T_1 + T_2 + \cdots + T_n$

$= T_n + T_{n-1} + T_{n-2} + \cdots + T_0$

$= \sum_{r=0}^{n} T_{n-r}$

$$\boxed{\sum_{r=0}^{n} T_r = \sum_{r=0}^{n} T_{n-r}}$$

If :- $\frac{1}{({}^{10}C_0)^2} + \frac{1}{({}^{10}C_1)^2} + \cdots + \frac{1}{({}^{10}C_{10})^2} = p$

$\frac{1}{({}^{10}C_1)^2} + \frac{4}{({}^{10}C_2)^3} + \frac{9}{({}^{10}C_3)^2} + \cdots + \frac{100}{({}^{10}C_{10})^2} = q$

(a) $\sum_{r=0}^{10} \frac{r}{({}^{10}C_r)^2}$ (b) $\sum_{0}^{10} \frac{r^3}{({}^{10}C_r)^2}$

(a) $\sum \frac{r}{({}^{10}C_r)^2} = \sum \frac{10-r}{({}^{10}C_r)^2} = \sum \frac{10}{({}^{10}C_r)^2} - \sum \frac{r}{({}^{10}C_r)^2}$

$2S = 10 \sum \frac{1}{({}^{10}C_r)^2}$

$2S = 10P$

$\boxed{S = 5P}$

(b) $\sum_{r=0}^{10} \frac{r^3}{({}^{10}C_r)^2}$; $\sum \frac{r^2}{({}^{10}C_r)^2} = q$, $\sum \frac{1}{({}^{10}C_r)^2} = p$

$\Rightarrow \sum \frac{(10-r)^3}{({}^{10}C_r)^2} \Rightarrow \sum \frac{10^3 - r^3 - 3\times 10^2 r + 3\times 10 r^2}{({}^{10}C_r)^2}$

Que: $\sum_{r=0}^{n} (r+1)(r+2)({}^nC_r)^2$

$\Rightarrow \sum_{r=0}^{n} (r^2+3r+2)({}^nC_r)^2$

Que: $\sum_{r=0}^{10} \frac{1}{{}^{10}C_r} = P$

find $\sum_{r=0}^{10} \frac{r}{{}^{10}C_r}$

$\Rightarrow \frac{0}{{}^{10}C_0} + \frac{1}{{}^{10}C_1} + \frac{2}{{}^{10}C_2} + \frac{3}{{}^{10}C_3} + \cdots$

$\Rightarrow \frac{10-10}{{}^{10}C_{10-10}} + \frac{10-9}{{}^{10}C_{10-9}} + \frac{10-8}{{}^{10}C_{10-8}} + \frac{10-7}{{}^{10}C_{10-7}} + \cdots \frac{10-0}{{}^{10}C_{10-0}}$

$= \sum_{r=0}^{10} \frac{10-r}{{}^{10}C_{10-r}} = \sum \frac{10-r}{{}^{10}C_r}$

$P = \sum \frac{10}{{}^{10}C_r} - \sum \frac{r}{{}^{10}C_r}$

$S = \sum \frac{10}{{}^{10}C_r} - S$

$2S = \sum_{r=0}^{10} \frac{10}{{}^{10}C_r}$

$2S = 10P$

$\boxed{S = 5P}$ ans.