10.5 Distance of a Point From a Line

The distance of a point from a line is the length of the perpendicular drawn from the point to the line. Let L : Ax + By + C = 0 be a line, whose distance from the point P (x_1, y_1) is d. Draw a perpendicular PM from the point P to the line L (Fig10.19). If

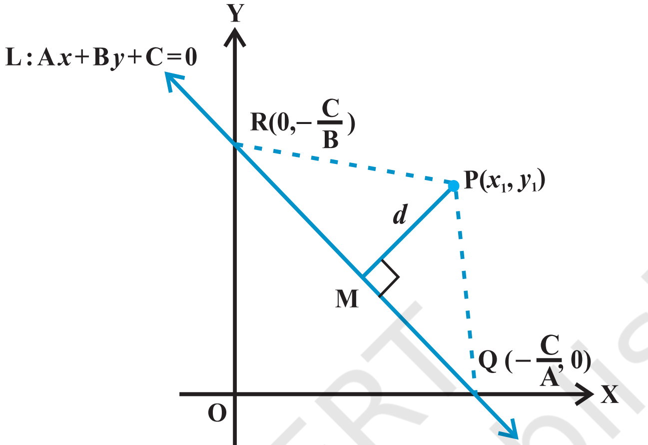

Fig10.19

the line meets the x-and y-axes at the points Q and R, respectively. Then, coordinates of the points are $Q\left(-\frac{C}{A},\ 0\right)$ and $R\left(0,\ -\frac{C}{B}\right)$. Thus, the area of the triangle PQR is given by

$$\text{area }(\Delta PQR) = \frac{1}{2} PM.QR \text{, which gives } PM = \frac{2\ \text{area }(\Delta PQR)}{QR} \qquad \ldots (1)$$

$$\text{Also, area }(\Delta PQR) = \frac{1}{2}\left| x_1\left(0+\frac{C}{B}\right) + \left(-\frac{C}{A}\right)\left(-\frac{C}{B} - y_1\right) + 0\left(y_1 - 0\right)\right|$$

$$= \frac{1}{2}\left| x_1\frac{C}{B} + y_1\frac{C}{A} + \frac{C^2}{AB}\right|$$

$$\text{or} \quad 2\text{ area }(\Delta PQR) = \left|\frac{C}{AB}\right| . \ \left|Ax_1 + By_1 + C\right|, \quad \text{and}$$

$$QR = \sqrt{\left(0+\frac{C}{A}\right)^2 + \left(\frac{C}{B} - 0\right)^2} = \left|\frac{C}{AB}\right| \sqrt{A^2+B^2}$$

Substituting the values of area (ΔPQR) and QR in (1), we get

$$\text{PM} = \frac{|\text{A}x_1 + \text{B}y_1 + \text{C}|}{\sqrt{\text{A}^2 + \text{B}^2}}$$

or $$d = \frac{|\text{A}x_1 + \text{B}y_1 + \text{C}|}{\sqrt{\text{A}^2 + \text{B}^2}}.$$

Thus, the perpendicular distance (d) of a line Ax + By+ C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|\text{A}x_1 + \text{B}y_1 + \text{C}|}{\sqrt{\text{A}^2 + \text{B}^2}}.$$

10.5.1 ***Distance between two parallel lines*** We know that slopes of two parallel lines are equal. Therefore, two parallel lines can be taken in the form

$$y = mx + c_1 \qquad \dots (1)$$

and $$y = mx + c_2 \qquad \dots (2)$$

Line (1) will intersect x-axis at the point A$\left(-\frac{c_1}{m},\ 0\right)$ as shown in Fig10.20.

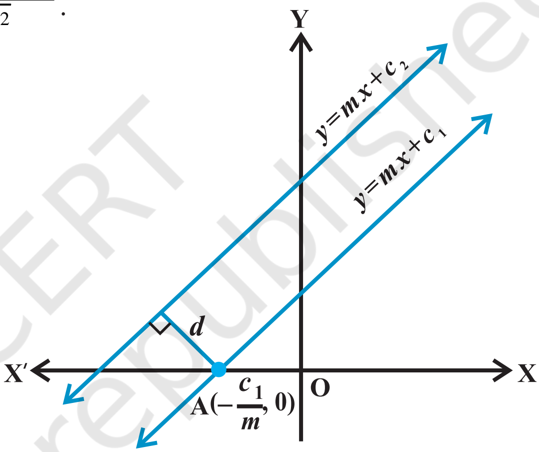

Fig10.20

Distance between two lines is equal to the length of the perpendicular from point A to line (2). Therefore, distance between the lines (1) and (2) is

$$\frac{\left|(-m)\left(-\frac{c_1}{m}\right) + (-c_2)\right|}{\sqrt{1+m^2}} \quad \text{or} \quad d = \frac{|c_1 - c_2|}{\sqrt{1+m^2}}.$$

Thus, the distance d between two parallel lines $y = mx + c_1$ and $y = mx + c_2$ is given by

$$d = \frac{|c_1 - c_2|}{\sqrt{1+m^2}}.$$

If lines are given in general form, i.e., Ax + By + C_1 = 0 and Ax + By + C_2 = 0, then above formula will take the form $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

Students can derive it themselves.

Example 18 Find the distance of the point (3, – 5) from the line $3x - 4y - 26 = 0$.

Solution Given line is $3x - 4y - 26 = 0$ ... (1)

Comparing (1) with general equation of line $Ax + By + C = 0$, we get

$$A = 3, B = -4 \text{ and } C = -26.$$

Given point is $(x_1, y_1) = (3, -5)$. The distance of the given point from given line is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|3.3 + (-4)(-5) - 26|}{\sqrt{3^2 + (-4)^2}} = \frac{3}{5}.$$

Example 19 Find the distance between the parallel lines $3x - 4y + 7 = 0$ and

$$3x - 4y + 5 = 0$$

Solution Here A = 3, B = –4, $C_1 = 7$ and $C_2 = 5$. Therefore, the required distance is

$$d = \frac{|7 - 5|}{\sqrt{3^2 + (-4)^2}} = \frac{2}{5}.$$

EXERCISE 10.3

1. Reduce the following equations into slope - intercept form and find their slopes and the y - intercepts.
   (i) $x + 7y = 0$, (ii) $6x + 3y - 5 = 0$, (iii) $y = 0$.
2. Reduce the following equations into intercept form and find their intercepts on the axes.
   (i) $3x + 2y - 12 = 0$, (ii) $4x - 3y = 6$, (iii) $3y + 2 = 0$.
3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.
   (i) $x - \sqrt{3}y + 8 = 0$, (ii) $y - 2 = 0$, (iii) $x - y = 4$.
4. Find the distance of the point (–1, 1) from the line $12(x + 6) = 5(y - 2)$.
5. Find the points on the x-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.
6. Find the distance between parallel lines
   (i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$ (ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$.