

If the term free from x in the expansion of $\sqrt{x} - \frac{k}{x^2}$ is 405, find the value of k .

Solution:

$$\text{Given } \sqrt{x} - \frac{k}{x^2}$$

From the standard formula of T_{r+1} we can write given expression as

$$\begin{aligned} T_{r+1} &= {}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{-k}{x^2}\right)^r = {}^{10}C_r (x)^{\frac{1}{2}(10-r)} (-k)^r x^{-2r} \\ &= {}^{10}C_r (x)^{5-\frac{r}{2}-2r} (-k)^r = {}^{10}C_r x^{\frac{10-5r}{2}} (-k)^r \end{aligned}$$

For the term free from x we have

$$(10 - 5r)/2 = 0$$

Which implies $r = 2$

So, the term free from x is

$$T_{2+1} = {}^{10}C_2 (-k)^2$$

$${}^{10}C_2 (-k)^2 = 405$$

$$\frac{10 \times 9 \times 8!}{2! \times 8!} (-k)^2 = 405$$

$$45k^2 = 405 \Rightarrow k^2 = 9 \therefore k = \pm 3$$