4) If M is a 3×3 matrix, where $\det M = 1$ and $MM^T = I$, where 'I' is ~~a~~ an identity matrix, prove that $\det(M-I)=0$

[2004 - 2 Marks]

Solution:

Given: $MM^T = I$, where M is a square matrix of order 3 and $\det M = 1$.

Now $\det(M-I) = \det(M - MM^T)$ $\quad [\because MM^T = I]$

$= \det[M(I - M^T)]$

$= (\det M)(\det(I - M^T))$

$[\because |AB| = |A||B|]$

$= -(\det M)(\det(M^T - I))$

$= -[\det(M^T - I)]$ $\quad [\because \det(M) = 1]$

$\Rightarrow \det(M-I) = -\det(M-I)$

$[\because \det(M^T - I) = \det[(M-I)^T] = \det(M-I)]$

$\Rightarrow 2\det(M-I) = 0 \Rightarrow \det(M-I) = 0$