

17. Let  $A = [a_{ij}]$  be a  $3 \times 3$  matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-1} & \text{if } i < j \\ 2 & \text{if } i = j \\ (-1)^{i+j} & \text{if } i > j \end{cases}$$

then  $\det(3 \operatorname{Adj}(2A^{-1}))$  is equal to \_\_\_\_\_.

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Solution: Since, the matrix  $A$  is,

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow |A| = 4$$

$$\therefore \det[3 \operatorname{Adj}(2A^{-1})] = \cancel{3 \cdot 2^2 \operatorname{adj}(A^{-1})} \left| 3 \cdot 2^2 \operatorname{adj}(A^{-1}) \right|$$

$$= 12^3 |\operatorname{adj}(A^{-1})| = 12^3 |A^{-1}|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = \boxed{108}$$