

2) Without expanding, show that $\Delta = \begin{vmatrix} \operatorname{cosec}^2 \theta & \cot^2 \theta & 1 \\ \cot^2 \theta & \operatorname{cosec}^2 \theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$

Solution: Applying $C_1 \rightarrow C_1 - C_2 - C_3$, we have

$$\Delta = \begin{vmatrix} \operatorname{cosec}^2 \theta - \cot^2 \theta - 1 & \cot^2 \theta & 1 \\ \cot^2 \theta - \operatorname{cosec}^2 \theta + 1 & \operatorname{cosec}^2 \theta & -1 \\ 0 & 40 & 2 \end{vmatrix} = \begin{vmatrix} 0 & \cot^2 \theta & 1 \\ 0 & \operatorname{cosec}^2 \theta & -1 \\ 0 & 40 & 2 \end{vmatrix} = 0$$