Adjoint of a Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Then $\operatorname{adj} A$ = Transpose of $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Here, A_{ij} is the cofactor of the element a_{ij}.

Theorem-1: If A be any given square matrix of order n, then

$$A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A|\, I$$

where I is the identity matrix of order n.

A square matrix A is said to be singular if $|A| = 0$.
A square matrix A is said to be non-singular if $|A| \neq 0$.

Theorem-2: If A and B are nonsingular matrices of the same order, then AB and BA are also nonsingular matrices of the same order.

Theorem-3: The determinant of the product of matrices is equal to product of their determinants, that is, $|AB| = |A||B|$, where A and B are square matrices of the same order.

Theorem-4: A square matrix A is invertible if and only if A is nonsingular matrix